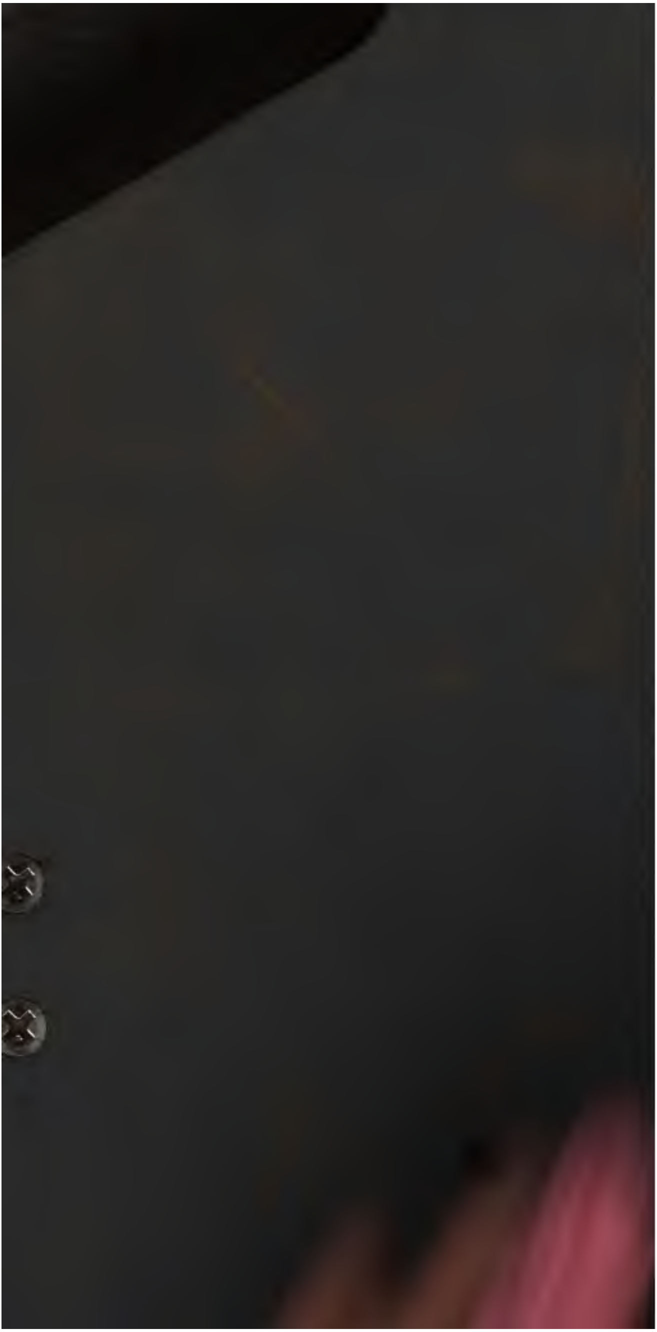

J. S. Abbot Jr

Jos. Abbot Jnr.

ARITHMETIC,

THEORETICAL AND PRACTICAL;

WHEREIN THE

FUNDAMENTAL PRINCIPLES

OF

THAT SCIENCE ARE EXPLAINED,

IN A

NEW, PERSPICUOUS AND FAMILIAR MANNER,

ESPECIALLY

NUMERATION, VULGAR AND DECIMAL FRACTIONS, THE RULE OF THREE BOTH SINGLE AND COMPOUND, &c.

DESIGNED TO ASSIST

BOTH INSTRUCTORS AND LEARNERS IN COMMUNICATING AND ACQUIRING A CORRECT KNOWLEDGE OF THE THEORY AND PRACTICE OF THIS IMPORTANT BRANCH OF MATHEMATICS.

PART I.

BY VICTOR VALUE,
Principal of Mantua Academy, near Philadelphia.

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J. G. Abbot Jr

tient of the *less* dividend is the *multiplicand* of the corresponding number in multiplication. Thus, (page 38) in division, No. 18, the quotient of the greater dividend, 1867776, divided twice by 4, is the less dividend, 116736, of the same No. 18—and the *quotient* of this less dividend, 116736, divided four times by 4, is 456, the multiplicand of the corresponding No. 18 of multiplication (page 24). This plan is followed to the end of the fourth set; and by it, we consult the convenience of teachers, save the time generally consumed in reviewing the operations, and render it difficult, not to say impossible, for learners to wrong themselves, by trying to impose on their instructors.—The same contrivance is applied to multiplication and division of decimals.

We have, under every rule, left some questions unanswered, to use scholars to depend on their own abilities; for it is observed that "pupils accustomed to work such questions only, as have the answers given, seem utterly at a loss when desired to work those to which they are not given."

The questions on multiplication and division, 1st and 2d sets, will be found very useful. They perfect the student in the multiplication table, and lead him to acquire readiness and accuracy in understanding and answering questions.

I have separated the multiplication table into two parts, to render the committing of it less arduous to the young learner.

The questions which immediately follow each part, have the same tendency; but multiplying and dividing several times in succession by the same number, for instance, by 2, 4, or 6, &c. (pages 24 and 37) will be found the best mode of making the multiplication table familiar to the students. A number of eminent teachers have successfully used both exercises.

I strongly urge the adoption of the mode of performing division explained and exemplified in the appendix,(page 161,) at least for beginners.

GEOMETRICAL RATIOS are introduced to make proportions more easily understood (page 59).

PROPORTIONS, as applicable to the rule of three, are, next to numeration, deserving the particular attention of learners; they should, therefore, endeavour to make themselves perfectly masters of their principles and properties, before they proceed further. As the first rules are founded on *numeration*, so, nearly all the subsequent ones depend on the properties of *proportions*.

A PROPORTION, from its nature and definition, is composed of four terms (page 61). In order that the student should never lose sight of this, I have invariably represented the unknown term by x (pages 67, 69, &c.) in stating the rule of three; and that x is placed as *first* term instead of *last;* for, it appears more natural to proceed from the first to the last, than as heretofore, from the last to the first; and, it seems, in consequence, to be more readily understood by the student. The method of beginning by the fourth term is, however, detailed, for those who prefer it; and in the appendix (page 166,) will be found the old method.

The word HOMOGENEOUS is introduced into the rule of three, as expressing the relations of the terms to one another, more accurately than any other expression would do. If it be thought a *hard word,* substitute for it *similar.* It is well, however, to remark that ~~*any word is easily remembered, pro*~~*vided its meaning is definite and well understood.*

By making the student observe that in the rule of three the terms are given *by pairs,* and by exercising him to distinguish them accurately (page 67,) the statement is much facilitated; because, each pair forming a *ratio,* he can immediately perceive, that a term *which has no homogeneous quantity* with which it may be *paired,* must be *disregarded.* The understanding of this is of considerable importance; for those odd terms are generally very perplexing(page 73). The plan of connecting the terms by *pairs* is *new,* and will, it is believed, prove useful.

Questions to be asked when the Class is engaged in stating.

Does the question require pounds, length, breadth, yards, &c.?—Then put down x pounds, x length, &c. &c.

What is its homogeneous term?—What is the other pair of terms?—On which of them does the question depend?

If on the *greater,* ask: will *more* require more or less?

If on the *smaller;* will *less* require more or less?

Place the ratio according to the last answer.

When the unknown term is unrepresented, the imperfect state of the proportion must necessarily produce an inaccurate impression on the student's mind. Having no fixed point on which to rest his attention, he can apply the reasoning, required in stating the question, only in a vague, loose, and unsatisfactory manner, very little calculated to lessen the difficulty he has to contend with. But, when the answer is

represented by x, that x becomes a determinate object, to which all the reasoning is appropriately and instantly applied; so that, nothing vague and uncertain can perplex the student's operations, and retard his progress.

This must be enumerated among the advantages attending the use of x; an advantage far from being unimportant, for it removes the obstacle which abstractions invariably present to youth who possess, but very imperfectly, the power of generalizing.

FRACTIONS (page 80).—VULGAR FRACTIONS, usually so difficult, intricate and repulsive, are treated in so plain and familiar a way, that they are within reach of every capacity: I have not hesitated, therefore, to present fractions thus early; for an intimate acquaintance with them will be found extremely useful.

The regular transition from addition to subtraction, multiplication and division, has so long been practised, and is consequently, so strongly impressed on our minds, from early associations, that it will probably appear preposterous to find multiplication and division of fractions treated of before addition and subtraction; and yet, these two last operations, requiring a preparation in which multiplication is necessary, I have thought it more consistent to teach *multiplication first*. By that means the pupil is made acquainted with the different operations in their proper succession; whilst in the usual manner, he is, in order to reduce to a common denominator, required to perform multiplication of fractions, before he is shown how it is to be done. No doubt, the perplexity many pupils experience on that subject, arises from that cause.

In cloth measure (page 111,) I have, (to follow other assistants,) made the French ell equal to 6 quarters, although, in commercial transactions, it is counted only as 5.

An *aune* or French ell equals in Paris feet, 3 feet, $7\frac{2}{3}$ inches, or $43\frac{2}{3}$ French inches; equal to $46\frac{1}{2}$ English inches; so that a French ell contains 1 yard, or 36 inches English, and $10\frac{1}{2}$ inches over; a quantity which is considerably less than half a yard, or 18 inches; consequently, a French ell ought not to be counted as 6 quarters, or $1\frac{1}{2}$ yards.—It is, in fact, only $1\frac{1}{2}$ English inches more than 5 quarters, or 45 inches, and in commercial transactions, it is rated at only 5 quarters; and 100 French ells are counted as 125 yards, although they are, in fact, equal to $129\frac{1}{6}$ yards.

COMPOUND DIVISION (page 151,) is not treated in the usual way.—In the first place, the divisor is not separated into se-

ARITHMETIC.

1. Arithmetic is a science which teaches to form, to express, to read numbers, and to perform on them various operations, to attain a proposed end.

2. A number is the collection or union of several unities: and the *unity* or *unit* represents any object whatever, considered as a whole, viz. a house, a book, a picture, an army, &c.

3. The first number is the *unit*; and all the others are formed by successively adding the unit to itself. Thus: one unit added to another, forms the particular number called *two:* if to the latter we add a unit, we have the number called *three,* and so on; so that, by adding a unit to the preceding collection, we may arrive at any number whatever.

4. But as it would be extremely difficult to recollect every individual or separate character, had one been invented to represent each particular number. The Arabs, to whose ingenuity we owe our system of *numeration,* adopted ten characters viz. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, called *figures,* by means of which, every possible number can be expressed. The first of these characters called *nought, cipher* or *zero,* represents nothing by itself, or has no signification; the others, have each a particular value, and are, therefore, called *significant figures.*

NUMERATION.

5. *Numeration* teaches how to represent all numbers by means of the ten characters just mentioned; to do this:

6. Numbers are divided into units of different classes; the value of each class is ten times that of the preceding one, counting from right to left.

7. So, with 10 units of the 1st class, has been formed a *single* unit of the second, called 1 *ten* of *units,* or simply 1 *ten:* with 10 units of the 2d class, or 10 tens, has been formed a single unit of the 3d, called *hundred* of *units* or simply *hundred:* with ten hundreds, or 10 units of the 3d class, has been formed a single one of the 4th class, called *units* of *thousands* or simply *thousand:* with 10 thousands or 10 units of the 4th class, has been formed a single unit of the 5th class, called

1 *ten* of *thousands:* with 10 tens of thousands or 10 units of the 5th class, has been formed a single unit of the 6th class, called 1 hundred of thousands; and so on, the 7th class being called units of millions, the 8th, tens of millions, the 9th, hundreds of millions, &c.

8. To elucidate this still more, let us:

To	1	to	2	to	3	to	4	to	5	to	6	to	7	to	8
Add	1		1		1		1		1		1		1		1
We have	2		3		4		5		6		7		8		9

each being expressed by a simple and different character. Now, if to 9 units, we add one, the number is called *ten;* but as there is no single character to represent that number, it is agreed to consider those ten units of the *first* class, as making only 1 of the *second,* called 1 ten. But, if set down by itself, that 1 will occupy the first rank or place; how then are we to make it fill the second? By putting the character 0, which has no value of itself, to the right of the figure 1, which will then be in the second place, whilst the 0, or nothing, will fill up the first or unit place, thus:

9	10	11	12	13
1	1	1	1	7
10	11	12	13	20

The first of these representing 1 ten and no unit. So that the same character 1, which represents the first number or unit, when placed in the first rank, represents 1 *ten,* or a quantity ten times greater, when placed in the second rank. If to 10, we add 1 unit, we form the number *eleven,* represented by 1 ten and 1 unit; let us add 1 more, we then have the number *twelve* or 1 ten and 2 units, and so on, till we come to 1 ten and 10 units; these 10 units being converted into 1 ten, we then have 2 tens and no unit. Since the figure 2 is to represent 2 tens, it must occupy the second place, of course *zero* is to fill up the first. Ex.

In *seventeen,* how many *tens?*
1 ten and 4 units form what number?
1 ten and 7 units form what number?
1 ten and 9 units do.
How many tens in twenty?
How many tens in sixteen? and also how many units?
How many units in eight?

Or the pupil may be directed to write down, eighteen—fifteen—fourteen—eleven—nine—sixteen.—Or else: 1 ten and 6 units—2 units and one ten—2 units and no ten—2 tens and no unit—twelve—nineteen—thirteen.

Exercises of this sort should be reiterated until the pupil has a sufficient knowledge to proceed.

9. We may successively obtain 3, 4, 5, 6, 7, 8, 9 tens, each particular one being represented by a single character; but when we reach 10 tens, as we have no single character to represent them, how shall they be expressed?—By considering those 10 tens are together, as worth only 1 single unit of the third order called 1 *hundred*. That 1 must then occupy the third place (from the right); of course, the first and second places are to be filled by two zeros thus: 100.

20	30	40	50	60	70	80	90
10	10	10	10	10	10	10	10
—	—	—	—	—	—	—	—
30	40	50	60	70	80	90	100

Instead of saying	2 tens, we say	twenty—of	6 tens	sixty,
	3 tens,	thirty,	7	seventy,
	4	forty,	8	eighty,
	5	fifty,	9	ninety.

How is the number composed of 6 tens called?—do. of 3 tens?—of 3 tens and 4 units? of 4 tens? of 4 tens and 6 units? of 5 tens? of 5 tens and 1 unit? of 6 tens and 2 units? of 7 tens? of seven tens and 7 units? of eight tens? of eight tens and 3 units? of nine tens? of nine tens and 9 units?

Write down thirty—thirty-three—thirty-eight—twenty-six—twenty-two—twenty-nine—forty—forty-one—forty-seven—sixty—sixty-nine—forty-five—seventy—seventy-six—seventy-three—eighty—eighty-two—ninety—ninety-one—and other exercises on any number inferior to one hundred.

In fifty-four, how many units? how many tens?—Why does the 4 represent units? Why does the 5 express tens? What would the 5 express, if it were in the first place?—In 54, invert the order of the two figures, what number will that produce?—and why?—&c. &c.

In forty how many tens? how many units?—&c.

In fifty-three?—in thirty-two?—forty-four?—fifty?—fifty-nine?—seventy?—seventy-three?—five?—eighteen?—seventy-seven?—one?—eighty-one?—ninety-five?—ninety-nine?—twenty-eight?—eighteen?

10. Thus we see that the figure 1 at the first place ex-

presses 1 unit; that the same figure 1, at the second place, expresses 1 ten, or a quantity ten times greater than 1 unit; at the third, it expresses 1 hundred, or a quantity ten times greater than the preceding or a hundred times greater than the first. The same thing takes place with the figure 2, which, when it occupies the 1st, 2d or 3d place, expresses 2 units, 2 tens or 2 hundreds, 222.—The same variations will occur with 3, as 333 : with 4, as 444 : with 5, as 555 ; with 6, as 666 : with 7, as 777 : with 8, as 888 : with 9, as 999. This last number called nine hundred and ninety-nine, contains 9 hundreds, 9 tens and 9 units.

EXERCISES.

In one hundred, how many tens? how many hundreds? how many units?—In one hundred and twenty-five, how many hundreds? how many units? how many tens? Why is it one hundred? why 5 units? why 2 tens?—In two hundred and twenty-seven, how many units? how many tens? how many hundreds?—and why?

In 604, how many units, tens, hundreds and why?

772—do.—450—201—666—382—328—347—102—210
832—do.—823—507—609—906—120—283—238—750

Why do you write 604 with 6, 0 and 4? Because there are 6 hundreds, no tens, and 4 units. Why do you write 102 with 1, 0 and 2?—do. 75?—94?—49?—509?—905?

What number is written with one 2, one 0 and one 8? Write down three hundred and seventy—four hundred and ninety—seven hundred—seventy—five hundred and nine—nine hundred and five—nine hundred and fifty—six hundred—hundred and four—six hundred and forty—four hundred and six—four hundred and sixty.

11. If, in the number 999, to the 9 units we add 1, the sum will be 10 units. These being converted into 1 ten, and added to the 9 we already have in the number at top, 10 tens will be formed. These 10 tens are equal to one hundred. Let us add it to the 9 we have above, and we then obtain 10 hundreds. With these 10 hundreds, we compose one single unit of the 4th class, called 1 *unit of thousands*. That 1, must then be at the 4th place from the right and consequently the 1st, the 2d and the 3d places are filled by three zeros.

4th	3d	2d	1st class
.	9	9	9
.	.	.	1
.	.	1	0
.	1	0	0
1	0	0	0

We here perceive that the figure 1, at the fourth place, expresses a quantity one thousand times greater than it does at the first.

In 1025, how many units? how many thousands? how many tens? hundreds?

2400—do.—4912—6073—5893—3958—1205—9214
7063—do.—8953—8385—2004—9142— 637—3589
9001—do.—6014—4006—1205—5901—1009.

Why in 1025, does the 1, express one thousand? the 5, five units?—the 2, two tens? the 0, no hundred?—The same questions on the other numbers.

What number is written with one 3, two 0 and one 7?
do. do. one 7, one 3 and two 1?
do. do, one 6, and three 9?
do. do. four 7?

Write down in figures one thousand, one hundred and eleven—one thousand two hundred and forty-three—three thousand four hundred and twenty-one—seven thousand—seven thousand and 9 units—seven thousand three hundred and two—eight thousand and twenty-five—nine thousand and seven—three hundred and five—three thousand three hundred and five.

12. When, by successive additions, we get 10 units of thousands or 10 units of the 4th class, they are converted into 1 single unit of the 5th class, called 1 *ten of thousands*; the 1st, 2d, 3d, and 4th places being filled by ciphers. 10 tens of thousands or 10 units of the 5th class, become, by a similar conversion, 1 single unit of the 6th class, called 1 *hundred of thousands*, represented by 100,000. Ten hundreds of thousands, 6th class, will form 1 single unit of the 7th class, called 1 *unit of millions*. 10 units of millions, give 1 single *ten of millions*; 8th class. 10 tens of millions, 1 single *hundred of millions*; 9th class.*

7th	6th	5th	4th	3d	2d	1st class
.	.	.	1	0	0	0
.	.	.	9	0	0	0
.	.	1	0	0	0	0
.	.	9	0	0	0	0
.	1	0	0	0	0	0
.	9	0	0	0	0	0
1	0	0	0	0	0	0

EXAMPLES.

In 12475, how many thousands? hundreds? tens of thousands? units? tens?

* See 1st remark, in appendix.

24164—do.—71209—200000—4109025—7528463
60008—do.—91270—314206— 712049—1000504
42056—do.—63008—741007—8000246— 504100
50625—do.—80047—777888—6240080— 400105.

Write down one million—three millions four hundred thousand—twenty-two millions, seven hundred forty thousand, six hundred and 4 units, One hundred eighty-nine thousand, six hundred and forty units.

13. Here we must observe that in the annexed table, each name belongs invariably to the class to which it is joined. Exercises,

What is the name of the 1st class on the right? that of the 2d? of the 3d? of the 4th? the 7th? the 9th?—the 6th? the 8th? the 5th? &c.

of millions			of thousands			of units			
hundreds	tens	units	hundreds	tens	units	hundreds	tens	units	
9th	8th	7th	6th	5th	4th	3d	2d	1st	class

What class comes on the right of the millions? of the hundreds of thousands? of the tens of thousands? of the hundreds? of the thousands? of the tens?

What class is called hundreds?—tens?—thousands?—units?—tens of thousands?—millions?—hundred of thousands? hundred of millions?

What class comes on the left of the hundreds?—of the units? of the thousands? of the millions? of the tens of thousands? of the tens? of the hundred of thousands? of the hundred of millions?

We must also notice that the words *units, tens* and *hundreds*, after being applied to the first three classes, are likewise employed, in the same succession, to name the three following classes; the only variation is found in the last word which becomes *thousands*, instead of *units*: as, *tens of thousands*, instead of *tens of units: units of thousands*, instead of *units of units*, or merely units: *hundreds of thousands*, instead of *hundreds of units*. In the next three classes, the same succession of *units, tens* and *hundreds* is again used, but the word *million* is joined to them instead of either *units* or *thousands*, and so on of all the other classes connected three by three.

14. This last observation naturally leads us to the division of any number whatever into parcels or periods of three classes each, or into *triads*,* beginning at the right hand and going to the left, (see examples). The first right hand *triad*, is the *triad of units*, which consists of three classes, viz: units, tens, and hundreds, of units, taken collectively and expressed by 0, 1, 2, or rather by 210. The second triad is that of the thousands, consisting likewise of units, tens and hundreds. The third triad is that of the millions, &c. &c.

triad of millions			triad of thousands			triad of units		
8	7	6	5	4	3	2	1	0
hundreds	tens	units	hundreds	tens	units	hundreds	tens	units

15. When you start from the units and ascend, the order of the triads is invariably the one just mentioned, viz: units, thousands, millions, &c. The order is exactly reversed, if beginning at the highest order, you descend; as, millions, thousands and units. Each triad, except the first one on the left, must always have three figures, one for the hundreds, one for the tens, and the other for the units.

EXERCISES.

When going to the left, what is the triad next to the units? to the thousands? to the millions?—In descending: what is the triad next to the thousands? to the millions? to the units?

16. To express or write down a number composed of any quantity of figures, it is sufficient to know how to write a number composed of three figures or a triad, which, as already seen, is composed of three classes; hundreds, tens, and units.

17. As soon as the quantity of hundreds is mentioned, write it down; do the same with the tens and likewise with the units.

EXERCISES.

Write down three hundred and sixty-nine—one hundred and twenty-six—seven hundred and fourteen—two hundred and thirty-five—four hundred and fifty-eight—five hundred and forty-three—nine hundred and eighty-one—eight hundred and seventy-five—seven hundred and ninety-eight; and any other of three significant figures.

If hundreds are given, and no tens, and no units, two ci-

* Triad. We select the English word *triad*, because it means the union of 3, or 3 united.

phers must be used to fill their places; so that the figure representing the hundreds should occupy its own place, viz: the third.

EXERCISES.

Write down: eight hundred—three hundred—five hundred—two hundred—one hundred—seven hundred—six hundred—nine hundred—four hundred—ten hundred.

If hundreds and units are given, and no tens; the hundreds and units are set down in their respective places, the 3d and 1st; and 0, in the 2d, to take the place of the tens; for, should that 0 be omitted, the figure intended to represent the hundreds, would be in the 2d, instead of the 3d place, and could not, of course, express hundreds or units of the 3d class.

EXERCISES.

Write down: three hundred and four—five hundred and nine—six hundred and seven—eight hundred and one—nine hundred and six—seven hundred and two—four hundred and three—two hundred and five—one hundred and eight—five hundred—five hundred and forty—six hundred—six hundred and sixty—nine hundred and five—three hundred and ninety—one hundred and seventy—one hundred and seven.

18. Now, as any number, however great, can be divided into triads, each one composed alike of hundreds, tens and units (except the first on the left, which may contain only units, or both tens and units); it is evident that when we are able to write one of these triads, we can write any number of them, each separately, and as it were, alone. We must merely attend to the invariable order in which they ought always to succeed each other, and be careful to put three zeros, for every triad, which is entirely left out, and one zero, for every *class* left out in any of the particular triads. Some examples will render this plain. If directed to write down, *seventy-four thousand and forty-five.*

In seventy-four, there are 7 tens and 4 units; of course, I set down 74; but they are to be thousands. The triad of thousands being the second, the triad of units is then to be placed on the right hand of the 74 thousands. I am directed to write only forty-five units, that is, 4 tens and 5 units, without any hundreds; 0, must then take the place of the wanting hundreds, and the number, seventy-four thousand and forty-five, will then be expressed by 74,045.

19. Had I been directed to write down *seventy four millions and four units,* I would first set down 74, and since they are

millions, the triad of thousands and that of units must be placed at the right hand of 74: but as there is no mention of thousands, the places of hundreds, tens and units of that triad must be filled up by 3 zeros; the number assumes then the following form: 74000. The triad of units is still to be placed; 4 units only are given; of course, I put zero for the hundreds, zero for the tens and have finally the number 74,000,004 in which it is easily seen that the whole triad of thousands is represented by three ciphers.

20. If instead of the preceding number, I had been directed to write down 704 millions and 4 units: after putting down 704 millions, observing that no thousands are given, I put three ciphers for that triad, and completing, as in the last example, the triad of units, I have 704,000,004 the proposed number.

Write down regularly under one another in figures, seven thousand, four hundred and twenty-three—seventy-two thousand, six hundred—six hundred fifty-three thousand, four hundred and twelve—three hundred and twenty-four thousand—twenty-two thousand, three hundred and fifty—seven millions, three hundred and four thousand, six hundred and nine—five millions—thirty-seven millions, forty-five thousand and seven units—six hundred millions, seven hundred and five thousand, and forty-two units—six hundred and forty-seven millions, eight hundred thousand, and seventy-eight units—seventy millions, three hundred and nine units—five millions and three units—eight millions and four units—seven hundred thousand and twenty-three units—seventeen millions and three hundred and forty units—nine hundred and four millions, six hundred and twenty-two thousand, one hundred and seven—four hundred millions, seven thousand and twenty-three units. One thousand millions—twenty thousand millions twenty-four units.

21. To read a number it must be divided from the right to the left into triads, by a comma, or a dot, and then beginning by the left, each triad must be read separately, as if it were alone, pronouncing at the end of each, the peculiar name it bears; as millions, thousands, &c. for instance, if the number 67654842 is given, beginning at the right, and dividing it into triads, I find the triad of units, thousands and millions; then going to the left triad, I say 67 millions, 654 thousands and 842 units. Read 7423—72106—653412—324000—22350—37045007—98600705042—647000800078—1004702—167000402589—1234567809—6901—72600—30003.

How a Number can be made 10, 100 or 1000, &c. times greater or smaller.

22. First, to make it 10, 100, 1000, &c. times greater, for instance, add on its right hand, 1, 2 or 3 zeros, &c. By adding 1 zero, the figure which before that, was at the right hand, and, of course, expressed units, will now, being at the 2d place, express tens, that is, a quantity ten times greater. By adding 2 zeros, the same figure instead of units will express hundreds, a quantity a hundred times greater, &c.

2
2 0
2 0 0
2 0 0 0

Second to make it 10, 100, 1000 &c. times less, take from the right 1, 2, 3, &c. zeros or figures. When you cut off the right hand character, the 2d one, which, before, expressed tens, becoming the 1st, expresses no longer tens, but units, that is, a quantity ten times less. When you cut off 2 characters, the 3d becoming the 1st expresses units instead of hundreds.

1 4 0 0
1 4 0
1 4
1 6 4 8
1 6 4
1 6

ADDITION.

23. Addition is an operation by which several numbers of the same kind are added together so as to form a single one equal to them all. The result of this operation is called, *sum, total* or *amount.*

24. To make an addition, the given numbers must be written under one another, so that the units correspond to the units, the tens to the tens, the hundreds to the hundreds, &c. when thus arranged, a line is drawn under the lowest number. Begin the operation by adding together the figures of the right hand column, do the same with the next one and so on to the last: the amount of each, if not above 9 units is placed under; but when greater than 9, that is, when it contains units and tens, set down only the units or right hand character, and keep the tens, or left hand figure, to be added or carried to those of the next class; under the last column set down the units of the amount and place the tens to the left.

25. The sum thus obtained is evidently equal to all the given numbers, since it is the amount of the units of all the classes which compose those given numbers.

Why do you under each column set down only the units, and keep the tens to be added to the following class?

26. Because, in the system of numeration 10 units of one class make but one of the following class to the left, consequently 1 is to be carried for each ten found in the amount.

NOTE. It is important to convince the young pupil of the necessity of classing together the units of the same kind; and that may be done in the following manner: If you have,

1 book 1 inkstand and 1 pen.	Can you say they make together, 3 books, 3 inkstands, or 3 pens? No, because they are all objects of different

kinds, which cannot be summed up together.

But if you have 1 book on the table, 1 book on the bench, and 1 book on the floor.	Can you not add them together, and say, you have 3 books? Yes, because they are all objects of the same kind.

Hence the necessity of placing together the quantities which are alike, that is, units with units, tens with tens, hundreds with hundreds, &c. &c.

I have to add together 49750, 586894, 68572, 4601, 7400, and 925321. 1st I place those given numbers under each other so that the units of each should be in the 1st column to the right, the tens in the 2d, and so on; when they are all down and underlined, I begin, at top, as follows: 0 and 4=4; 4 and 2=6; 6 and 1=7 and 1=8. The number 8 is then the quantity of units contained in all the given numbers, and as in 8 units there is no ten, I set down 8 under the 1st row, and proceed to the row of tens, saying: 5 tens and 9=14 and 7=21 and 2=23 tens, or 3 tens and 2 hundreds; these 2 hundreds being units of the 3d class, it would be incorrect to place them in the 2d, therefore out of the 23, I merely set down the 3 tens under the 2d row, and keep the 2 hundreds to add them to those of the next left hand column. This is evidently the same as setting down the units of each partial sum and carrying the tens it contains to the next superior column; for, when I consider 23 as a partial sum, or 23 by itself, 3, expresses units, and 2, tens. Since I carry 2 from the 2d, column to the 3d, I say, 2 and 7=9 and 8=17 and 5=22 and 6=28 and 4=32 and 3=35 hundreds or 5 hundreds, which I place under, and 3 thousands which I keep to carry to the row of thousands: this is again setting down the units and carrying the tens of the partial amount 35.—I go on summing up every column until I come to the last, which,

$$\begin{array}{r} 49750 \\ 586894 \\ 68572 \\ 4601 \\ 7400 \\ 925321 \\ \hline 1,642,538 \\ \hline \end{array}$$

including the 2 carried from the preceding one, gives 16—I place the 6 units under that last column and the 1 ten on its left.—The grand total is 1,642,538.

EXAMPLES. 1st *Series.*

Add twenty, twenty-one, twenty-two, twenty-three, and twenty-four.

Add twenty-five, twenty-six, twenty seven, twenty-eight and twenty-nine.

Add one hundred and thirty-four—one hundred and thirty-three—one hundred and thirty-two—one hundred and thirty-one—one hundred and thirty.

Add two-hundred and thirty-nine—two hundred and thirty-eight—two hundred and thirty-seven—two hundred and thirty-six—two hundred and thirty-five.

Add	340—	342—	344—	341—	343
"	345—	348—	347—	346—	349
"	454—	452—	453—	451—	450
"	459—	458—	457—	456—	455
"	560—	561—	564—	563—	562
"	567—	565—	569—	566—	568
"	674—	673—	672—	671—	670
"	679—	678—	677—	676—	675
"	781—	783—	788—	789—	780
"	787—	782—	784—	783—	786
"	894—	893—	892—	891—	890
"	899—	898—	897—	896—	895
"	2990—	3992—	4993—	5994—	6995
"	7996—	8997—	9998—	9991—	9999

2d Series.

Ex. 1st	26435	2d	43004	3d	854321	4th	12428
	7089		2008		67890		7564
	12478		9006		432109		152135
	48325		4081		8756		719004
	———		500009		9786534		82000
	94327		———		———		———
			558108				

5. What is the sum of twenty-three thousand, four hundred and ninety-six—four thousand, six hundred and two—seven hundred and thirty-nine thousand and five units—six thousand, nine hundred and eighty-four—ninety-six units—two hundred thousand and forty-five units? Ans. 974,228.

6. Add together 76—49204—816—1820—243567—6789—9678. Ans. 311,950.

7. Washington was born in the year one thousand seven hundred and thirty-two, he was, when he died sixty-seven years old, in what year did he die? Ans. 1799.

8. Franklin was born in 1706, he was eighty-four years old when he died, in what year did his death occur? Ans. 1790.

9. If from Philadelphia to Lancaster there are sixty-six miles, from Lancaster to Swatara twenty-four—from Swatara to Harrisburg ten—thence to Carlisle seventeen—from Carlisle to Stony-creek one hundred and eight, from there to Greensburg forty, and from Greensburg to Pittsburg thirty-one; how many miles are there from Philadelphia to Pittsburg? Ans. 296.

10. Sir Isaac Newton was born in the year 1642, he lived 85 years, in what year did he die? Ans. 1727.

11. Gun-powder was invented in 1330; 111 years afterwards printing was discovered; 51 years after that, America was discovered by Columbus; 330 years have elapsed since that, in what year are we? Ans. 1822.

12. St. Petersburg is 30 degrees E. long. from London. Philadelphia is 75° W.—What is their difference in longitude? Ans. 105°.

13. Cape Horn is about 57 degrees S. lat. Philadelphia is 40° N.—What is their difference in latitude? Ans. 97°.

14. Voltaire was born in 1693, he lived 85 years, what year did he die? Ans. 1778.

15. Rome was founded 753 years before Christ, how long is it from that to the year 1822. Ans. 2575.

16. Coriolanus was banished from Rome 491 years before Christ, how many years is it since that banishment took place? Ans. 2313.

17. From Portland to Boston there are 116 miles—from Boston to Hartford 97—from there to New York 109—from New York to Trenton 57—from Trenton to Philadelphia 30. How far is Portland from Philadelphia? Ans. 409 miles.

SUBTRACTION.

27. Subtraction is an operation by which a number is taken from another of the same kind. The result of the operation is called, *excess*, *remainder* or *difference*.

28. To make a subtraction, place the smaller number under the other; units corresponding to units, tens with tens, &c. draw a line below, and beginning on the right hand, take

each figure in the lower line from the figure above it and set down the remainder, if any, or zero if there be none.

29. But it may happen that the figure below is greater than the one above it, in such a case, as it cannot be subtracted, observe the following directions.

1st. Borrow 1 from the next higher class on the left.

2d. Consider that 1 as 10, and add them to the upper figure, from which alone you cannot take the lower one.

3d. Take the lower figure from this *amount* and set the difference down.

4th. When you come to the upper figure from which you have borrowed, count it as one less, and take the lower one from it, if possible; if not, borrow from the next left hand class, and proceed as just now directed, taking care, always to count every figure from which you have borrowed as 1 less. Examples will make this plain.

```
1 6 2 5 8 7 1
  4 3 4 9 5 0
-------------
1 1 9 0 9 2 1
-------------
```

After placing the two given numbers under one another, units under units, tens under tens, &c. and underlining them, beginning at the units place, and considering the upper number 1625871 as what I possess, out of which I must pay 434950, I say: 0 units from 1, 1 remains, which remainder I set down below; 5 tens from 7—2 are left—I likewise set 2 down; 9 hundreds from 8, cannot be subtracted, then I

1st. Borrow 1 from the next upper class on the left, (i. e.) from 5 thousands.

2d I count that 1 thousand as 10 *hundreds*, and adding them to the 8, I have 18 hundreds.

3d. From that amount, 18, I take off 9, and set down the difference 9.

4th. Coming now to the 5 from which I have borrowed 1, I count it only as 4, and say: 4 thousand from 4* nothing remains, set down 0.

In the next column we find 3 tens of thousands to be taken from 2. This cannot be done; I then have recourse to the next class, and borrow 1 from the 6 hundreds of thousands; I convert it into 10 *tens of thousands*, and after adding them to the 2 I have already, from the sum 12 I subtract 3—the excess 9 being put down, I proceed to the following column,

* Some, instead of lessening the *upper* figure, add *one* to the *lower* figure. In that case the *upper* one is not altered. They would, in the worked examples, say 5 from 5, instead of 4 from 4. The remainder is evidently the same.

and considering the 6 from which I have borrowed 1, as equal only to 5, I say 4 from 5, 1 remains—and, as in the last column there is nothing to subtract from the 1 million, I set it down and thus terminate the operation. So that after paying what I owe, I shall yet have an excess of 1190921.

N. B. Let the pupil place a dot over every figure from which he borrows.

30. The 1 borrowed is converted into 10, because, as already taught in numeration, 1 unit of any class is always equal to 10 units of the next inferior class, or of the next right hand class—of course, by merely borrowing 1, the subtraction can always be effected, since the lower figure can never exceed 9.*

31. But when you want to borrow 1 from the left hand figure, should it be a zero, as you cannot borrow there, since 0 is worth nothing, what is to be done?

In this case you borrow 1 from the first *significant* figure you can find, and converting that 1 into 10, you proceed as before directed; but, observe that the zero or zeros intervening must be counted as 9, when you subtract from them.

```
Ex. 5 0 0 0 4 0 0 2
      9 6 0 4 7 3 1
    _______________
    4 0 3 9 9 2 7 1
```

The 2 numbers being properly placed and underlined, I say 1 unit from 2, 1 remains: 3 tens from 0, cannot be subtracted, I must then borrow 1 from the hundreds, but as there is none, I go on to the thousands, and from the 4 borrowing 1, I convert it into 10 hundreds. Shall I now carry these 10 hundreds to the class of tens? No, for 1 alone of these hundreds will answer my purpose: I, of course, leave 9 in the class of hundreds, (the 0 is then changed into 9), and converting the 1 wanted into 10 tens, I take the lower 3 from them, and set down the remainder 7: now, I have 7 hundreds to take, not from 0, but from 9, 2 remain. Having borrowed 1 from the upper 4, there remain only 3, I then say 4 thou-

* Some take the under figure from 10, and to the remainder adding the upper figure, they set down the sum. In the worked example, when acting on the hundreds, they would, instead of taking 9 from 18, as we did, say 9 from 10, one remains, and adding the upper figure 8 to that one, they would set down their sum 9.

These different methods are all equally good in practice, but as they do not all offer the same facility of demonstration, we have selected the above, thinking that the principles on which it is founded, are simple, natural, and will be easily understood by the student.

Let the teacher who adopts a different one from ours, explain its principles to his pupils.

sands from 3, I cannot. It becomes necessary to borrow; but as there is no tens of thousands, no hundreds of thousands, no millions, I have to borrow from the 5 tens of millions. That 1 ten of millions is equal to 10 millions: as *one* of these 10 millions will answer my present purpose, I leave 9 with the 1st left hand 0; I, afterwards, convert the 1 million into 10 hundreds of thousands, leaving 9 of them with the 2d 0, I change the borrowed 1 into 10 tens of thousands, leaving 9 of them with the 3d 0, I change the 10th into thousands, adding the 3 I already have; from 13, the amount, I take the 4 below and set down the excess 9. Each 0 having assumed the value of 9, the successive remainders are 9, 3, 0 and finally 4, since I borrowed 1 from the 5.*

32. The result of this operation is evidently the difference of the 2 numbers, since it expresses the difference of their units, of their tens, and in general, the difference of all the classes which compose them.

EXAMPLES.

From	46	3451	10000	38	443678	74525
Take	34	374	8405	17	233534	61816
Difference	12	3077	1595			

3842	7435	74016	45000	30	77005
2898	270	18047	38	14	4778

1st	24605	2d	306	3d	47894	4th	1000	5th	3666
	4505		85		8275		7		789

* Those, who, instead of lessening the upper figure, add one to the lower, must consider each intervening 0 as 10, instead of 9.

Federal money may serve to remove the difficulty, if any remain, respecting the changing of 0 into 9. Suppose I want to pay 3 cents, and that to do it I have only 1 eagle. The 2 numbers will be

1 0 0 0 3 9 9 7	Having no cents, it is evident I cannot pay the 3 I owe; but I can change my eagle for 10 dollars—I am unable yet to pay the 3 cents. Shall I now change those 10 dollars into dimes? No; that would be useless, for by changing only 1 into 10 dimes I shall have more

than enough to pay my debt, consequently, I shall have 9 dollars left. Must I now change my 10 dimes into cents? No; by changing only 1 for 10 cents, I shall have enough to acquit my debt; of course, 9 dimes will remain in my possession. Out of my 10 cents, paying the 3 I owe, 7 remain to me. Owing no dimes, I shall keep the 9 I received in exchanging. Owing no dollars, 9 will remain in my possession. It is easily seen how each 0 is transformed into 9.

6. From 246 take 97. Ans. 149. From 4001 take 3998. Ans. 3. From 7062 take 5953. Ans. 1109.

7. From six thousand two hundred and thirty-five, take three hundred and forty-one. Ans. 5894

8. From one million, take twenty-two thousand and seven units. Ans. 977993.

9. America was discovered in 1492 by Columbus, we are in 1822, how many years has it been known? Ans. 330.

10. How many years since the discovery of printing in 1441, to this present year 1822. Ans. 381.

11. The city of Mexico is about 100° W. long. from London. Philadelphia being at 75° what is the difference?

12. New-York is 87 miles from Philadelphia—and Boston 289—how many miles farther is Boston?

13. From Philadelphia to Washington there are 135 miles, and 96 to Baltimore, how many miles from Baltimore to Washington?

14. Sir Isaac Newton was born in 1642, he died in 1727. How long did he live?

15. Lancaster is 66 miles from Philadelphia and Pittsburg 296—how much farther is Pittsburg?

16. Washington was born in 1732, he died in 1799, how old was he then?

17. Suppose Baltimore contains 50,084 inhabitants, Boston 43296—Charleston 32065--N. Orleans 36,000--New-York 122314—Philadelphia 118253. Salem (Mass.) 15294. Trenton 5426. Savannah 7,000—Richmond 14,000—Providence 12485. Hartford 6,000—will the population of all those cities be equal to that of London, computed at 1,000,000 of inhabitants?—If not, what is the difference?

18. And likewise what is the difference between the population of all those places, and that of Paris, computed at 720000.

19. The Declaration of Independence was published the 4th of July, 1776—how many years have elapsed since that time to the 4th of July, 1820.

ADDITION AND SUBTRACTION.

How to Prove Addition and Subtraction.

33. To prove an addition, sum up all the numbers except the top line, and subtract the 2d amount from the 1st, the dif-

ference must evidently give the top line, since it was not included in the 2d amount.

Another proof is sometimes made as follows.

34. Begin a new addition by the left hand column, and without setting down that 2d sum, subtract it from the one previously obtained, and already placed under that same column, note down the difference; it is evident that this difference becomes as many tens, when united to the figure placed under the next column to the right, from this, subtract the amount obtained by the 2d addition of that next column, set down that 2d difference, consider it as tens, when prefixed (mentally,) to the next right hand figure, subtract again the new sum, and go on to the units, the last remainder must obviously be 0, since from the whole, you successively take every part of which it was composed.

EXAMPLE.

The amount of the annexed addition being 931485, to prove it, I proceed thus: I add the 1st column on the left and find 6; without setting it down, I subtract it from the 9, found under the same column by the previous addition, and set down the remainder 3. This, I consider as 3 tens, which being united (mentally,) to the figure 3, placed under the next right hand column, gives 33, from which I subtract 31, found by re-adding the figures of that 2d row; the remainder is 2; I place it under, and uniting it as before to the figure 1, above, I have 21, from which I take 20, found by the re-addition of the 3d row; the difference 1, when united to the following 4 gives 14; the re-addition of the 4th row giving 14, I have 0 for difference—the next remainder is 1, and as under the column of the units I find 0 for remainder, I conclude the 1st amount is correct.

```
  4 6 7 5 2
  9 4 4 0 0
  9 8 3 2 9
6 9 2 0 0 4
-----------
  9 3 1 4 8 5
-----------
  3 2 1 0 1 0
```

1st 743251	2d 743106	3d 415274	4th 123456
631204	14429	96852	210987
20917	359987	162749	345678
162736	23456	14287	432109
			567890

35. To prove a subtraction add the *difference* to the *less number;* the sum must be equal to the *greater number;* since the difference is the excess of the superior number over the

inferior; if you add up those two component parts, you must necessarily find the whole.

EXAMPLES.

5 0 0 0 4 0 0 2	If, in the annexed subtraction, I add to
9 6 0 4 7 3 1	the less number, 9,604,731, the difference,
	40,399,271, the amount is 50,004,002, equal
4 0 3 9 9 2 7 1	to the superior number.
5 0,0 0 4,0 0 2	

1st. 4 5 6 7 0 0	2d 2 0 0 0 5 0	3d 5 8 3 2 1 0 5 4
6 7 8 9 1	1 3 8 4 5 2	5 7 3 4 7 2 5 3

1. Take 29 from ninety-four. Ans. 65.

2. What is the difference between 105 and 87? 18.

3. From four thousand, take four hundred, what remains? 3600.

4. Charles borrowed of John 579 dollars, and paid him at one time 120 dollars, at another 370, what remains due to John? 89.

5. A boy who had 600 nuts, eat 54, gave his brother 216, sold 128, lost 79, what number had he left? 123.

6. If, in a city having a population of eighty-five thousand souls, 1512 males, and 1474 females die every year; but if at at the same time 1845 males, and 1795 females are born annually, what will be the increase of population at the end of 3 years, and the number of the inhabitants of the city? 86,962.

7. In four bags are 560lb. in the 1st there is 96lb. in the 2d 124—in the 3d 58, how much is there in the 4th? 282.

8. A boy receives from his father 200 cents, from his mother 185—from his aunt 75—from his sister 6; he buys apples for 31 cents, nuts for 20—a penknife for 316—how much has he yet? 99.

9. A vintner or wine merchant bought 30 casks of brandy, containing 3127 gallons, and sold 18 casks, containing 1871 gallons; how many casks and gallons had he left?

12 casks and 1256 galls.

10. A man was born in the year 1785—how old is he in 1820?

11. My grocer sends me his bill amounting to 83 dollars; but for cotton sold him he owes me 71 dollars, for rice 24,

for bees-wax 47—am I in his debt, or is he in mine, and for how much?

12. How much time has elapsed since the conquest of England, by William, duke of Normandy, in A. D. 1066, to the year 1820.

13. Baltimore contains fifty thousand and eighty-four inhabitants, Salem (Mass.) fifteen thousand two hundred and ninety-four, Trenton, five thousand four hundred and twenty-six. Are there as many inhabitants in those 3 cities as there are in Philadelphia, which contains one hundred and eighteen thousand, two hundred and fifty-three? What is the difference?

14. Mary went shopping, and bought articles to the amount of twelve dollars, and gave a twenty dollar note, what change must she have?

15. John bought goods amounting to two hundred and forty-six dollars, and gave a five hundred dollar note in payment, what change must he receive?

16. Bought marketing, viz. beef for fifty-six cents,—butter for thirty-seven cents,—eggs for twenty-five cents,—vegetables for twelve cents; and gave a five dollar note in payment: I desire Miss Ann to tell what money I must receive in change? $3 70.

17. Maria goes shopping, and buys muslin for six dollars, lace for four dollars, and a shawl for ten dollars; what change must she have out of a fifty dollar note?

18. Mount Ætna (in Sicily) is ten thousand, nine hundred and fifty-four feet above the level of the sea; and Mount Vesuvius (in Italy) three thousand, nine hundred; how much is the one higher than the other? 7054.

19. Mount Chimboraso, the highest part of the Andes (in S. America) is twenty thousand, six hundred and eight feet above the level of the sea; and Mount Blanc, the most elevated part of the Alps, fifteen thousand three hundred and three feet; what is the difference? 5377.

20. The Peak of Teneriffe, in one of the Canary Islands, is thirteen thousand two hundred and sixty-five feet above the level of the sea; and Mount Heckla, in Iceland, is five thousand; the difference is required. 8,265.

21. The first book was printed with moveable types cut out of wood, invented by Guttemburg, an inhabitant of Mentz, in the year 1430; how many years is it since that time to 1822?

22. The first Bible was printed in the year 1462, how many years have since elapsed?

23. The sect of Friends appeared in the year 1651; how many years have since elapsed?

24. The ships of capt. Cook returned from circumnavigating the globe the first time, in the year 1771; how many years have since elapsed?

25. The city of London contains 1,000,000 inhabitants, and Paris seven hundred thousand; what is the difference?

26. Petersburg contains one hundred and seventy thousand people, and Stockholm 60,000; the difference is required?

27. Vienna has six hundred thousand people, and Amsterdam 214,000; what is the difference?

28. Edinburg contains 90,000 souls, and Dublin 156,000; what is the difference?

MULTIPLICATION.

36. MULTIPLICATION is an operation by which a number called multiplicand, is repeated as many times as there are units in another number termed *multiplier*. The result of that operation is called *product*.

The *multiplicand* and *multiplier* are called *factors* of the *product*.

MULTIPLICATION TABLE. 1*st part.*

Twice			3 times			4 times			5 times			6 times		
0	is	0	0	is	0	0	is	0	0	is	0	0	is	0
1		2	1		3	1		4	1		5	1		6
2		4	2		6	2		8	2		10	2		12
3		6	3		9	3		12	3		15	3		18
4		8	4		12	4		16	4		20	4		24
5		10	5		15	5		20	5		25	5		30
6		12	6		18	6		24	6		30	6		36
7		14	7		21	7		28	7		35	7		42
8		16	8		24	8		32	8		40	8		48
9		18	9		27	9		36	9		45	9		54
10		20	10		30	10		40	10		50	10		60
11		22	11		33	11		44	11		55	11		66
12		24	12		36	12		48	12		60	12		72

EXAMPLES ON MULTIPLICATION. 1*st Set.*

The pupil may be exercised in answering the following questions without setting them down.

Let him be informed that, by multiplication, the value of several objects is found, when the value of one is known. For instance: if one apple cost 2 cents, 2 apples will cost 2 cents twice, or 4 cents; three apples will cost 2 cents three times, or 6 cents, &c. and let him explain the solution of each question by a similar mode of reasoning.

What cost 3 apples at 2 cents a piece?

What cost 3 apples at 3 cents a piece?

What cost 4 yards tape at 2 cents a yard?

What cost 5 cakes at 2 cents a piece?

What cost 6 pints of cider at 2 cents a pint?

If 1 pear is worth 2 apples, how many apples can I have for 2 pears?

If a boy translate 3 pages in 1 hour, how many pages can he translate in 3 hours?

What cost 4 hats at $3 a piece?

What cost 5 pencils at 3 cents a piece?

What cost 6 glasses at 3 cents a piece?

In 1 quart there are 2 pints, how many pints in 2 quarts?
do. do. 7 do.?
do. do. 6 do.?
do. do. 4 do.?
do. do. 8 do.?

In 1 yard there are 3 feet, how many feet in 4 yards?
do. do. 10 do.?
do. do. 9 do.?

In 1 gallon there are 4 qrts. how many quarts in 2 gallons?
do. do. 8 do.?
do. do. 12 do.?
do. do 6 do.?

How many pints in 4 quarts or 1 gallon?

One league contains 3 miles, how many miles in 4 leagues? in 7?—in 8?—in 12?—in 9?—in 6?—in 5?

One yard is equal to how many feet?

One gallon to how many quarts?

One gallon to how many pints?

One dollar to how many quarters?

In 1 pint there are 4 gills, how many gills in 3 pints?
do. do. 8 do.?
do. do. 1 quart?
do. do. 2 pints?
do. do. 6 pints?

One penny is worth 4 farthings, how many farthings in 7 pence?
do. 6 do.?
do. 11 do.?
do. 12 do.?
do. 9 do.?

In 1 peck, there are 2 half pecks, how many half pecks in 4 pecks?
do. 6 do.?
do. 8 do.?
do. 7 do.?
do. 12 do.?

In 1 yard there are 6 times 6 inches, how many inches is that?

What cost 5 lemons at 5 cents a piece?

What cost 9 pounds veal at 5 cents a pound?

What cost 10 oranges at 5 cents a piece?

What cost 12 gallons molasses at 5 shillings a gallon?

What cost 8 flower pots at 5 cents a piece?

In one French ell there are 6 quarters, how many quarters in three French ells?

How many quarters in 9 French ells?

How many quarters in 12 French ells?

How many quarters in 8 French ells?

One dollar is worth 4 quarters, how many quarters in 5 dollars?—in 7?—in 12?—in 8?—in 3?—in 6?—in 9?—in 11?—in 10?

In one league how many miles?

In one quart how many pints?

One French ell is how many quarters?

One pint how many gills?

When the multiplier does not exceed 9, work by

RULE 1st.

37. Set the multiplier under the units of the multiplicand, underline them, and beginning at the units multiply one after another the figures of the multiplicand by the multiplier, set down under each class respectively, the units belonging to it, and, as in addition, carry the tens of each partial product to the next superior class. The whole of the *last* product must be set down.

Factors	Multiplicand	246704	
	Multiplier	5	placed under the units of multiplicand.
	Product	1233520	

After placing the factors as above, I say: 5 times 4 units give 20 units, or 2 tens and no units, I, consequently, place 0 under the first column, and keep the 2 tens to add to the product of the next superior class, 5 times 0 is 0—and I should put down 0, had I not the 2 tens from the 1st product to add here; after setting down the 2, I proceed to the hundreds, saying; 5 times 7 hundred give 35 hundred, or else 5 hundred, which I put down to their place, and 3 thousands, which must be carried to their own column, and so on to the last figure, 2, of the multiplicand, which being multiplied by 5, gives 10, and having added the 2 kept from the preceding product, I set down 12 as the final one, and obtain 1233520 for total product.

2d Set.

1. Multiply 210 by 2, six times in succession and give the 3d and 6th products.

2. Multiply 210 by 3, six times in succession and give the 3d and 6th products.

Factors	210	Multiplicands	210	Factors
	2	Multipliers	3	
	420		630	
	2		3	
	840		1890	
	2		3	
	1680	3d Products	5670	
	2		3	
	3360		17010	
	2		3	
	6720		51030	
	2		3	
	13440*	6th Products	153090	

3. Multiply 432 by 2, six times in succession, giving the 3d and 6th products.

4.	do.	543 by 2 do.	
5.	do.	432 by 3 do.	
6.	do.	543 by 3 do.	
7.	do.	210 by 4 do.	
8.	do.	432 by 4 do.	
9.	do.	543 by 4 do.	
10.	do.	210 by 5 do.	
11.	do.	432 by 5 do.	
12.	do.	543 by 5 do.	
13.	do.	210 by 6 do.	
14.	do.	432 by 6 do.	
15.	do.	543 by 6 do.	

16. Multiply 456 by 2, six times in succession, giving the 4th and 6th products.

17.	do.	456 by 3 do.
18.	do.	456 by 4 do.
19.	do.	456 by 5 do.
20.	do.	456 by 6 do.
21.	do.	765 by 4 do.
22.	do.	765 by 5 do.
23.	do.	765 by 6 do.
24.	do.	4837 by 2 do.
25.	do.	3487 by 3 do.
26.	do.	3487 by 4 do.
27.	do.	7459 by 5 do.
28.	do.	7459 by 6 do.

* The teacher may, if he think fit, cause the pupil to observe that by multiplying 210 by 2, three times in succession, it is the same as if he had multiplied it by 8; because $2\times2\times2=8$ That by multiplying it 6 times by 2, it is the same as multiplying by 64, &c.—And he may then take an opportunity of telling him, that sometimes instead of multiplying by a large number, its smaller factors are used.

There are 20 cents in a pistareen, how many cents are there in 5 pistareens?

There are 25 cents in a quarter of a dollar, how many cents in 4 quarters?

An eagle is worth 10 dollars, how many dollars in 3 eagles?

There are 6 feet in a fathom, how many are there in 11 fathoms?

How many in 10 fathoms?—in 9?—in 7?—in 8?—in 12?

There are 1760 yards in a mile, how many in 4 miles?

How many in 6?—in 2?—in 5?—in 3?

There are three hundred and sixty-five days in a year, how many in 4 years?

How many days in 6 years?—in 3?—in 5?—in 2?

There are 5280 feet in a mile, how many feet in 5 miles?

How many in 4 miles?—in 2?—in 6?—in 3?

There are 32 quarts in a bushel, how many quarts in 3 bushels?

How many in 5?—in 4?—in 6?—in 2?

One league is equal to 3 miles, the Isthmus of Darien being 20 leagues broad, how many miles is that?

How many miles in the strait of Dover which is 7 leagues broad?

The strait of Gibraltar which is 8 leagues?—The Isthmus of Corinth, which is 2 leagues?

MULTIPLICATION TABLE. *2d part.*

7 times		8 times		9 times		10 times		11 times		12 times	
0	is 0	0	is 0	0	is 0	0	is 0	0	is 0	0	is 0
1	7	1	8	1	9	1	10	1	11	1	12
2	14	2	16	2	18	2	20	2	22	2	24
3	21	3	24	3	27	3	30	3	33	3	36
4	28	4	32	4	36	4	40	4	44	4	48
5	35	5	40	5	45	5	50	5	55	5	60
6	42	6	48	6	54	6	60	6	66	6	72
7	49	7	56	7	63	7	70	7	77	7	84
8	56	8	64	8	72	8	80	8	88	8	96
9	63	9	72	9	81	9	90	9	99	9	108
10	70	10	80	10	90	10	100	10	110	10	120
11	77	11	88	11	99	11	110	11	121	11	132
12	84	12	96	12	108	12	120	12	132	12	144

In 1 gallon there are 8 pints, how many pints in 7 gallons?—in 5?—in 6?—in 9?—in 12?—in 11?

In Pennsylvania the dollar is divided into 8 eleven penny

bits, how many eleven penny bits in \$8 ?—how many in 3 ?—in 9 ?—in 10 ?—in 7 ?—in 12 ?—in 11 ?—in 6 ?

One square yard is equal to 9 square feet, how many sq. feet in 3 sq. yards ?—in 7 ?—in 4 ?—in 9 ?—in 8 ?—in 12 ?

In 1 dollar there are 10 dimes, how many dimes in \$4 ?—in 3 ?—in 7 ?—in 5 ?—in 9 ?—12 ?—8 ?—2 ?—6 ?—10 ?—11 ?

In the Eastern States one dollar is equal to 8 nine pences, how many nine pences are there in \$5 ?—in 7 ?—in 9 ?—in 11 ?—in 6 ? in 8 ? in 10 ? in 12 ?

What cost 3 pounds of sugar at 11 cents a pound ?
do. 6 quarts of beer at 11 cents a quart ?
do. 8 oranges at 11 cents a piece ?
do. 4 balls of twine at 11 cents a ball ?
do. 9 yards cloth at \$11 a yard ?
do. 5 gallons of brandy at 11 shillings per gallon ?
do. 10 dozen apples at 11 cents a dozen ?

If a man ride 7 miles in one hour, how many can he ride in 11 hours ?

There are 12 inches in 1 foot, how many inches in 3 feet ? how many in 5 ?—in 8 ? in 11 ? in 6 ? in 12 ? in 9 ? in 2 ? in 7 ? in 4 ? in 10 ?

In one peck there are 8 quarts, how many quarts in 6 pecks ? how many in 10 ? in 7 ? in 9 ? in 12 ?

One dime is worth 9 pence Penn. cur. how many pence in 2 dimes ? in 7 ? in 11 ? in 8 ? in 9 ? in 10 ? in 12 ?

29. Multiply 120 by 7, six times in succession, and give the 3d and 6th products,

30.	do.	432 by 7	do.
31.	do.	120 by 8	do.
32.	do.	432 by 8	do.
33.	do.	120 by 9	do.
34.	do.	120 by 10	do.
35.	do.	120 by 11	do.
36.	do.	120 by 12	do.
37.	do.	674 by 7	do.
38.	do.	647 by 8	do.
39.	do.	647 by 9	do.

40. Multiply 785 by 10, six times in succession, and give the 4th and 6th products.

41.	do.	785 by 11	do.
42.	do.	785 by 12	do.
43.	do.	879 by 7	do.
44.	do.	879 by 8	do.
45.	do.	879 by 9	do.
46.	do.	879 by 10	do.
47.	do.	879 by 11	do.
48.	do.	879 by 12	do.
49.	do.	6435 by 8,	3 times
50.	do.	4798 by 12,	3 do.

There are 20 shillings in one pound, how many shillings are there in 8 pounds how many in 6 ?—in 10 ? in 7 ? in 9 ? in 11 ? in 12 ? in 4 ? in 3 ? in 5 ?

There are 365 days in a year, how many days in 4 years ? how many days in 6 years ?—in 7 ?—in 10 ? in 12 ?—in 8 ?—in 9 ? in 11 ? in 3 ?

One hundred weight is equal to 112 pounds, how many pounds in 3 cwt.? how many in 4?—in 7? in 9? in 8? in 5? in 12? in 11? in 6? in 10?

In one half barrel are 16 gallons, how many gallons in 3 half barrels? how many in 2? in 4? in 5? in 6? in 7? in 12? in 11? 10? 9? 8?

One degree contains 60 geographic miles, how many miles in 6 degrees? how many in 3?—in 7?—in 2?—4? 5? 8? 10? 9? 12? 11?

There are 7 days in a week, how many days in 4 weeks? —in 12? in 5? in 7? in 2? in 11? in 9? in 8?

There are 2240 pounds in a ton, how many pounds in 3 tons? how many in 4? in 8? in 12? in 5? in 7? in 9? in 11? in 10? in 6?

In one bushel there are 32 quarts, how many quarts in 4 bushels? how many in 5 bushels? in 8? in 10? in 6? in 12? in 9? in 3? in 11?

One acre is equal to 4840 square yards, how many yards in 2 acres? in 3? in 4? in 5? in 6? in 7? in 8? in 9? in 10? in 11? in 12?

38. We are now sufficiently acquainted with multiplication to give our attention to some very important principles belonging to it. If I multiply

	2	4 and	6
by	2	2	2
	4	8	12

I obtain three products, 4, 8 and 12—the second of which is double the first, because the *second* multiplicand 4, is the *double* of the *first* multiplicand 2—and that ought to take place, because, from the definition of multiplication, which consists in *repeating the multiplicand as many times as there are units in the multiplier*, it is evident, that if, in a second operation, we have the *same* multiplier and a multiplicand *twice as great* as in the first, this double multiplicand being repeated as many times as the single one, the product will be double; *and its increase will always be the same as that of the multiplicand.* Hence, when we multiply 6 by 2 we find 12, a product 3 times as great as 4, because the third multiplicand 6, is 3 times as great as 2.

If now we reverse the case, and take a multiplicand *less* than before; $\frac{1}{2}$ or $\frac{1}{3}$ for instance; the product being this $\frac{1}{2}$ or $\frac{1}{3}$, (instead of the whole) repeated the same number of times, must be only the $\frac{1}{2}$ or $\frac{1}{3}$ of what it was before. Then we infer,

39. 1st That the *product increases or diminishes in the same proportion as the multiplicand.*

40. In the two following multiplications, we see that the second product 32 is the double of 16. The multiplicand 8, being the same in both cases, the alteration must be owing to the multiplier 4, which is twice as great as 2, and it ought to be so, because, when the multiplicand is the same, if the multiplier become double, that multiplicand being repeated twice as many times as before, the product, which expresses that number of times, will consequently be twice as great.

8	8
2	4
16	32

On the contrary, if the multiplier is less than before; $\frac{1}{2}$ or $\frac{1}{3}$ for instance, the same multiplicand being repeated, not as many times as before, but merely $\frac{1}{2}$ or $\frac{1}{3}$ of it, the operation must yield a product only $\frac{1}{2}$ or $\frac{1}{3}$ of the first. Should the multiplier be 10 times less than before, as the same multiplicand would not be taken as many times as at first, but on the contrary 10 times less, the product must necessarily be the tenth part of the preceding or be 10 times less. Hence we infer,

41. *2d That the product increases or decreases in the same proportion as the multiplier.*

42. Now since the product increases as the multiplicand and multiplier, and decreases as they do, from the two inferences above, we draw the general conclusion that: *the product depends* DIRECTLY *on its factors*, that is, increases or decreases as they do.

43. Until now we have spoken only of the increase of one of the factors, while the other remains unchanged; but, should they both increase or diminish in a ten fold proportion, or, in other words, become each 10 times greater or less, what will be the effect on the product?

44. The multiplicand being 10 times greater, the product will on that account be 10 times greater: the multiplier being also 10 times larger, will make the product 10 times more. Must we say that being 10 times greater on account of the multiplicand, and 10 times greater on account of the multiplier, the product will altogether be 20 times greater? Many scholars will at first conclude so, because 10 and 10 make 20; but on a little reflection they will find this conclusion erroneous, for the product is not formed by the *addition* of its factors, but by their *multiplication*. Hence, we must *multiply the increase of the multiplicand by that of the multiplier*, or 10 by 10, the product of which is 100.

EXAMPLES.

1	10	2	20
1	10	2	20
1	100	4	400

When the multiplier exceeds 9, that is, when it contains more than one figure work by

RULE 2d.

45. Multiply by each figure of the multiplier separately, first by the right hand one, then by the next and so on; placing their respective products one under another, with the right hand figure of each product directly under that figure of the multiplier by which it is produced. Add these partial products together, and their amount will evidently be the total product required; since it will be composed of the products of the multiplicand, by every individual class of the multiplier.

46. Why is the right hand figure of each product placed under the figure of the multiplier by which it is produced?

We will explain this in detailing the following operation. After having carefully placed the units, tens, &c. of the multiplier under the same classes of the multiplicand, and drawn a line below, I multiply all the multiplicand by the units of the multiplier; that is, by 3, and set down the product. The next figure of the multiplier is 3; not 3 *units*, but 3 *tens*, that is, a quantity 10 times greater; this second product must then be 10 times greater than the first, and to render it 10 times greater, I have only to make its right hand figure express tens instead of units; and that is done by placing it under the second column, or the column to which the multiplier belongs. The next figure of the multiplier is 0, and if I take the multiplicand 0 times, the product is 0, I set down 0 under the third class and proceed to the 4 thousands, a quantity one thousand times greater than units; the product by 4 must of course be 1000 times greater than if obtained by 4 units; its right hand figure must then be placed under the fourth column, or the column corresponding to the figure by which you actually multiply, and so on of every one.

```
       7 4 6 0 5 8 9
           2 4 0 3 3
   -----------------
     2 2 3 8 1 7 6 7
   2 2 3 8 1 7 6 7
 2 9 8 4 2 3 5 6 0
1 4 9 2 1 1 7 8
-------------------
1 7 9,3 0 0,3 3 5,4 3 7
-------------------
```

47. If both factors are terminated by ciphers, or only one of the factors, disregard the ciphers, and multiply at once by the first significant figure of the multiplier, but at the right hand of the total product, place as many ciphers as you left off in the factor, or factors.

48. By omitting one, two, or three ciphers you make the factors 10, 100, or 1000 times less than they really are; the product, depending on its factors, will of course be 10, 100, 1000, &c. times smaller than it actually ought to be; to give it its true value, you must then make it 10, 100, 1000, &c. times greater; and that is done by the addition of one, two, three, &c. ciphers; and, in a word, by the addition of as many as were neglected in the factor, or factors.

49. We have seen that the product increases and diminishes as its factors; but should the *multiplicand* of a second operation be 3 times greater than it was in the first, and the second *multiplier* be 3 times smaller than the first, what would be the effect on the product? Would it be greater or smaller than the first, or would it be equal to it? (See example.)

EXAMPLES.

1st	2d	
6	18	3 times greater
9	3	3 times less
54	54	products
42	7	6 times less
12	72	6 times greater
504	504	

It would be 3 times greater than the first, on account of the increase of the multiplicand, but the decrease of the multiplier rendering it 3 times less, the final result must necessarily be the same. We may then conclude that the product will experience no change when one of *its factors becomes a certain number of times greater, if, at the same time, the other becomes the same number of times less.*

50. On this principle is founded a manner of proving multiplication, which consists in taking the ½ of one of the factors, and doubling the other. The product of this second operation must evidently be equal to the first.

EXAMPLE.

8472948	4236474
9	18
76256532	76256532

51. The product must always be of the same denomination or kind as the multiplicand, or in other words, express the same things; since it is nothing but the multiplicand itself, repeated as many times as there are units in the multiplier. The multiplier is considered as an *abstract number*, that is, as a number which does not represent any particular object or kind.

52. By multiplication, is ascertained the price of several objects when we know the price of a single one. For, it must be evident that the value of all the objects will be equal to that of one, repeated as many times as there are objects, or as many times as there are units in the number of those objects, that is, in the multiplier; for instance, if for 1 orange you give 3 cents, for 2, you must give 3 cents twice—for 3 oranges, 3 cents 3 times; for 4, 3 cents 4 times; and in short the price must be repeated as many times as there are oranges or objects.

EXAMPLES. *3d Set.*

1.—	2.—	3.—	4.—
475964	237982	7482	2494
12	24	36	108
5711568			19952
			24940
			269352

5. Multiply	2212 by	18	16.	do.	25	"	13
6. do.	1106 "	36	17.	do.	125	"	36
7. do.	9864 "	27	18.	do.	1096	"	243
8. do.	4587 "	19	19.	do.	1529	"	57
9. do.	12000 "	50	20.	do.	1200	"	500
10. do.	760000 "	80	21.	do.	7600	"	8000
11. do.	5070 "	10	22.	do.	507	"	100
12. do.	75600 "	40	23.	do.	756	"	4000
13. do.	5426 "	608	24.	do.	10852	"	304
14. do.	85230 "	527	25.	do.	42615	"	1054
15. do.	6790 "	2008	26.	do.	3395	"	4016

27.—	28.—
764824	382412
257	514
5353768	1529648
3824120	382412
1529648	1912060
196559768	196559768

29. 1 ounce apothecary weight being equal to 24 scruples, how many scruples are there in 78 ounces?

30. 1 ton avoirdupois being equal to 2240 pounds, how many pounds in 89 tons?

31. If a vessel sail 89 miles in 1 day, how many miles will that be in a week or 7 days?

32. Bought 13 boxes of raisins, each weighing 28lb. neat, how many cents must I pay for the whole at 15 cents per lb.?

33. The sun goes 15° in 1 hour, how many degrees is that in 24 hours?

34. A degree contains 20 marine leagues, how many leagues in 90°—and since 1 league equals 3 geographic miles, how many miles in the same number of degrees?

35. A room is 22 feet long and 17 broad, how many square feet does it contain, or what is its surface in square feet?

The surface or area, or superficies of a rectilineal figure is ascertained by multiplying its length by its breadth,

36 How many square yards in a garden 68 yards long, and 57 broad?

37. How many square inches in a square board whose side contains 29 inches?

REMARK.—*In a square, the length is equal to the breadth, hence the above board is 29 inches long and 29 inches broad.*

38. There are 1440 minutes in a day, how many minutes in 1 week or 7 days?

39. One week containing 10080 minutes, how many minutes in one year or 52 weeks?

40. 1 pipe of brandy containing 126 gallons, how many gallons in 80 pipes?

41. A merchant bought 6 pieces of linen containing 25 yards each, 2 other pieces containing 24 yards each, and 1 more containing 26 yards. How many yards in the whole?

42. There are 12 chests of tea, each weighing 96 pounds: the chests which contain the tea weigh each 20 pounds: what does the tea weigh without the chests, and what is the price of the tea at 125 cents per pound? 114000 cents.

43. Multiply 342 by 22, and from the product subtract 428. 7096.

44. There are 10 bags of coffee weighing each 120 pounds, and 12 bags weighing each 135 pounds. What is the weight of the whole? 2820 pounds.

EXAMPLES. *4th Set.*

1.	Multiply	11154	by	16		
2.	"	5789	"	18		
3.	"	9864	"	27		
4.	"	2212	"	36		
5.	"	17359	"	54		
6.	"	3018	"	48,	and add 8 to the product.	
7.	"	21880	"	72		
8.	"	134297	"	63	"	18
9.	"	6288	"	87	"	73
10.	"	39023	"	96	"	43
11.	"	5874	"	83	"	22
12.	"	496	"	88	"	54
13.	"	3444	"	71	"	32
14.	"	94659	"	75	"	5
15.	"	234	"	15		
16.	"	587	"	19		
17.	"	865	"	23		
18.	"	679	"	28		
19.	"	26924	"	35		
20.	"	601	"	422		
21.	"	1323	"	3467		
22.	"	40058	"	342,	to the product add	16
23.	"	11214	"	120000		
24.	"	70000	"	1068	"	64000
25.	"	86440	"	10911	"	1003
26.	"	9416	"	7890	"	4984
27.	"	36000	"	46	"	24000
28.	"	37640	"	1234		

DIVISION.

53. DIVISION is an operation by which a number called DIVIDEND, is divided into as many parts as there are *units* in another number called DIVISOR; or, it may be defined, an operation by which we ascertain how many times the *divisor* is contained in the *dividend*. The result of the operation gives a number called *quotient*. The *quotient* then, marks how often the dividend contains the divisor.

By division is found, how many articles can be obtained for a certain sum, when the price of 1 article is known. For instance:

If one melon cost 3 cents, how many melons can I have

The object of Division is this — to find one of the factors of a product when the o
factor is given —

for 12 cents?—It is evident that I can buy as many times one melon, as 3 cents, the price of one melon, is contained in 12 cents, the money I wish to spend. Hence, I have to find how many times 3 is contained in 12, or to divide 12 by 3; the quotient 4, shows the number of melons.

The student should be exercised in answering the following questions, without setting them down, and in explaining them by a mode of reasoning similar to the above.

1st Set of Questions.

How many cakes at 1 cent a piece, can you buy for 3 cents? Reasoning.—I can buy as many times one cake, as the *price* of one cake goes into 3 cents, that is, one into 3; consequently the answer is 3, found in dividing 3 by 1.

How many cakes at 1 cent a piece, can you buy for 5 cents? for 7 cents?

How many apples at 2 cents a piece can you buy for 4 cents?*

How many can you buy for 6 cents? for 10 cents? for 12 cents?

How many yards of tape at 2 cents a yard can you buy for 8 cents?

A fruit woman sells her peaches at 2 cents a piece, how many of them can you buy for 10 cents?—how many for 18 cents? for 16 cents?

With 15 cents, how many pencils can you buy, at 3 cents a piece?—with 9 cents?†—with 12?—with 6?

A servant receives 18 cents to go and buy cider at 3 cents a pint, how many pints must she bring home?

If a boy can translate 3 pages in 1 hour, in how many hours can he translate 9 pages?—6 pages? 12 pages? 21 pages? 27 pages?

Jesse agrees to give Samuel 1 marble for 3 chesnuts, how many marbles must he give for 9 chesnuts? for 15 chesnuts? for 24? for 18?

If the price of 1 hat is $3, how many hats can I have for 12 dollars? for $18? for $27? for $36? for $30?

One yard being equal to 3 feet, how many yards in 7 feet? in 21? in 11 feet? in 12 feet? in 13 feet? in 14 feet?

One quart being equal to two pints, how many quarts in 8 pints? in 9 pints? in 12? in 15? in 16? in 19?

---

* To divide a number by 2, is the same as to take its half.

† To divide a number by 3, is the same as to take its third part.

One dollar being equal to 4 quarters, how many dollars must I receive for 16 quarters? for 24 qrs.? for 32 qrs.?*

One pint being equal to 4 gills, how many pints are there in 12 gills?—in 20 gills? in 8 gills? in 28? in 36?

One league being equal to 3 miles, how many leagues in 30 miles? in 21 miles? in 22? in 26? in 36? in 38?

The Strait of Gibraltar being 24 miles broad, how many leagues is that?

How many lemons at 5 cents a piece can I buy for 20 cents? for 10 cents? for 30 cents? for 40 cents? for 25? for 15? for 35?

If a butcher sells his veal at 5 cents a pound, how many pounds must he give me for 50 cents? for $37\frac{1}{2}$ cents? for 60 cents? for $62\frac{1}{2}$ cents?

One gallon being equal to 4 quarts, how many gallons are there in 20 quarts? in 36 quarts? in 40? in 24? in 48? in 34? in 23?

If 1 gallon of molasses cost 5 shillings, how many gallons can I buy for 25 shillings? for 15? for 45? for 60? for 15?

One French ell being equal to 6 quarters, how many French ells in 12 quarters? in 24 qrs.? in 60 qrs.? in 48? in 18?

When the divisor is contained in the dividend a certain number of times, and there is something over, that overplus is called a *remainder*.

54. To make a division, when the divisor does not exceed 9, place the dividend and divisor as in the annexed example, and see how often the divisor is contained in the first left hand figure or figures of the dividend,‡ put down the partial quotient under; if there be a remainder to this first division, consider it as tens, and, as such, add it to the next right hand figure of the dividend, see then, how often these contain the divisor, set down the quotient under and go on, in the same manner, always considering the remainder as tens, which must be joined to the next inferior class. Should there be no remainder to some of these partial divisions, find how often the divisor is contained in the next right hand figure of the dividend, and if it be not contained once in it, set down 0 under and see how often it is contained in that and the next right hand figure, and proceed as above directed until you have divided all.

---

* To divide a number by 4, is the same as taking the fourth of it—by 5, the fifth—by 6 the sixth, &c. &c.

‡ See note in appendix.

EXAMPLE.

Dividend
Divisor 6)37445310

Quotient 6240885

After placing the dividend and divisor as they are here; observing that the divisor 6, is not contained in 3, the the first left hand figure of the dividend, I take the first and second, 37, and finding that they contain 6, the divisor, six times and 1 over, I set down 6 for partial quotient. The remainder being 1 million, I convert it into 10 hundreds of thousands, and adding them to the 4 I have already, that gives 14 for a new *dividual*,* which contains 6, the divisor, twice and 2 over, I set down the second partial quotient 2. The remainder being 2 hundreds of thousands, I convert them into 20 tens of thousands, and 4 I have already, give 24 tens of thousands for a new *dividual*; this containing 6, the divisor, 4 times exactly, I put down that new quotient 4, and I examine whether or not, the next figure 5, of the dividend contains the divisor 6; as it does not, I set down 0 under the 5 thousands, and converting, them into 50 hundreds and adding the 3 I have already, (or what comes to the same thing, taking for a new dividual the figure 5 and the next one) I see how often 6 goes into 53; the quotient is 8 and the remainder 5. I then proceed and find the other partial quotients 8 and 5; and as the division is without remainder, I conclude that the dividend contains the divisor 6240885 times exactly.

Dividend
Divisor 5)62046

Quotient 12409+1 remainder

Here the dividend contains the divisor 12409 times and 1 unit over. To complete the quotient, place the last remainder to the end of it, above a line, and write the divisor below.

Complete quotient $12409+\frac{1}{5}$.

* *Dividual*, a portion of the dividend.

2d Set of Questions.

1. Divide 13440 by 2, six times in succession, and give the 3d and 6th quotients.

Divisors	Dividends	
1st	2)13440	
2d	2)6720	1st
3d	2)3360	2d
4th	2)1680	3d quot.
5th	2)840	4th
6th	2)420	5th
	210	6th quot.*

2. Divide 153090 by 3, six times in succession and give the 3d and 6th quotients.

Divisors	Dividends	
1st	3)153090	
2d	3)51030	
3d	3)17010	
4th	3)5670	3d quot.
5th	3)1890	
6th	3)630	
	210	6th quot.

3. Divide 27648 by 2, 6 times in succession, giving the 3d and 6th quotients.

			times
4. "	4344	by 2,	3
	34752	" 2	"
5. "	374928	" 3	"
	11664	" 3	"
6. "	395847	" 3	"
	14661	" 3	"
7. "	13440	" 4	"
	860160	" 4	"
8. "	27648	" 4	"
	1769472	" 4	"
9. "	34752	" 4	"
	2224128	" 4	"

			times
10. "	3281250	by 5,	6
11. "	54000	" 5,	3
	6750000	" 5,	3
12. "	8484375	" 5,	6
13. "	9797760	" 6,	6
14. "	20155392	" 6,	3
	93312	" 6,	3
15. "	25334208	" 6,	2
	703728	" 6,	4
16. "	29184	" 2,	3
	3648	" 2,	3
17. "	36936	" 3,	4
	332424	" 3,	2

* The pupil may be made to observe that by dividing 13440, by 2, three times successively, it is the same as if he had divided it by 8, because the 1st division gives him the 1-2 ; the 2d, gives him the 1-2 of the 1-2 or 1-4, and the 3d, gives him the 1-2 of the 1-4 or the 1-8 of the first number, and that to find the real number by which he has divided, he may multiply the several divisors together, their product will be the number wanted, &c.

18. "	1867776 "	4, 2 "
	116736 "	4, 4 "
19. "	285000 "	5, 4 "
	7125000 "	5, 2 "
20. "	590976 "	6, 4 "
	21275136 "	6, 2 "
21. "	3133440 "	4, 6 "
22. "	11953125 "	5, 6 "
23. "	35691840 "	6, 6 "
24. "	309568 "	2, 6 "
25. "	2542023 "	3, 3 "
	94149 "	3, 3 "
26. "	14282752 "	4, 3 "
	223168 "	4, 3 "
27. "	116546875 "	5, 3 "
	932375 "	5, 3 "
28. "	348007104 "	6, 6 "

20 cents making 1 pistareen, how many pistareens in 100 cents?

25 cents being equal to a quarter of a dollar, how many quarters in 100 cents?

Knowing that 1 eagle is worth 10 dollars, how many eagles in 30 dollars?

There are 6 feet in a fathom, how many fathoms in 66 feet? how many in 60? in 54? in 42? in 48? in 72?

How many yards in 1 mile, when in 4 miles there are 7040 yards? When in 6 miles there are 10560 yards?—in 2, 3520 yards?—in 5 miles, 8800? in 3 miles, 5280?

How many days in 1 year, when in 4 years there are 1460 days?—when in 6 years, there are 2190 days?—in 3 years, 1595 days? in 5, 1825? in 2, 730?

How many feet in 1 mile, when in 5 miles, there are 26400?—when in 4 miles, there are 21120 feet? in 2 miles, 10560 feet? in 6, 31680? in 3, 15840?

How many quarts in 1 bushel, when 3 bushels contain 96 quarts?—when 5 bushels contain 160 quarts? 4 bushels 128 quarts? 6 bushels 192? 2 bushels 64?

The Isthmus of Darien being 60 miles broad, how many leagues is that, when we know that it takes 3 miles to make 1 league?

The Strait of Dover being 21 miles broad, how many leagues is that?

The Strait of Gibraltar is 24 miles how many leagues is that?

How many in the Isthmus of Corinth which is 6 miles broad?

One gallon being equal to 8 pints, how many gallons in 56 pints?—in 40?—in 48?—in 72?—in 96?—in 88?

8 eleven penny bits, in Pennsylvania, making one dollar, how many dollars in 64 eleven penny bits?—in 24?—in 72?—in 80?—in 56?—in 96?—in 88?—in 48?

One square yard is equal to 9 feet square, how many

square yards in 27 square feet?—in 63?—in 36?—in 81?—in 72?—108?

For 33 cents, how many pounds of sugar can I buy at 11 cents a pound?

At 11 cents per quart of beer, how many quarts can I buy for 66 cents?

At 11 cents for 1 orange, how many can I buy for 88 cents?

For 44 cents, how many balls of twine can I buy at 11 cents per ball?

With $100 how many yards of cloth can I buy, at 9 dollars per yard?

In 1 dollar there are 10 dimes, how many dollars in 40 dimes? in 30 dimes? in 70? in 50? in 90? in 120? in 80? in 110?

In the Eastern states one dollar is equal to 8 nine-pences, how many dollars are there in 40 nine-pences? in 56? in 72? in 88? in 48? in 64? in 80?

One dime is worth 9 pence Pennsylvania currency, how many dimes in 18 pence? in 63? in 99? in 72? in 81? in 90? in 108?

With 55 shillings, how many gallons of brandy can I buy at 11 shillings per gallon?

With $1.10, how many dozen apples can I buy at 11 cents per dozen?

A man rode 77 miles in 11 hours, how much is it per hour?

There are 12 inches in 1 foot, how many feet in 36 inches? how many in 60? in 96? in 132? in 72? in 144? in 108? in 24? in 84? in 48? in 120?

29. Divide 14,117,880 by 7, six times in succession.

30. "	50,824,368	" 7	6
31. "	31,457,280	" 8	"
32. "	113,246,208	" 8	"
33. "	63,772,920	" 9	"
34. "	120,000,000	" 10	"
35. "	212,587,320	" 11	"
36. "	358,318,080	" 12	"
37. "	79,295,426	" 7	"
38. "	169,607,168	" 8	"
39. "	343,842,327	" 9	"

40. Divide 785,000,000 by 10, six times in succession.

41. "	1,390,675,385	" 11	6
42. "	2,348,997,440	" 12	"
43. "	103,413,471	" 7	"
44. "	230,424,576	" 8	"
45. "	467,136,639	" 9	"
46. "	879,000,000	" 10	"
47. "	1,557,202,119	" 11	"
48. "	2,624,679,936	" 12	"
49. "	3,294,720	" 8,	3
50. "	8,290,944	" 12,	3

If 160 shillings make 8 pounds, how many shillings in 1 pound?

If in 4 years there are 1460 days, how many days in 1 year?

If in 6 cwts. there are 672 pounds, how many pounds in 1 cwt.?

If 192 gallons make 12 half barrels, how many gallons in 1 half barrel?

If 480 geographic miles make 8 degrees, how many miles in 1 degree?

If in 9 weeks there are 63 days, how many days are there in 1 week?

If in 7 tons there are 15680 pounds, how many is that per ton?

If in 11 bushels there are 352 quarts, how many is that per bushel?

If in 10 acres there are 48400 square yards, how many is that per acre?

Light comes from the sun to us in 8 minutes, the distance from the sun is 96,000,000 of miles, how many miles is that per minute?

In 7425 square feet, how many square yards, when it takes 9 square feet to make 1 square yard?

55. If we perform the two following divisions, $2)\frac{4}{2}$ and $)\frac{8}{4}$, we find that the second quotient 4 is the double of the first. The divisor being the same in both cases, the variation must be caused by the change of the dividend, which is double, and it is evident that whenever the dividend increases, it will contain the same divisor a greater number of times than before, consequently the quotient must be greater.

56. When the dividend becomes less, it will not contain the same divisor as many times as before, and then the quotient must be less. Hence we infer, that:

57. 1st *The quotient* INCREASES *when the dividend increases, and* DECREASES *when the dividend decreases*, or in other words that, *the quotient depends* DIRECTLY *on the* DIVIDEND.

58. In the following divisions, $3)\frac{12}{4}$ and $6)\frac{12}{2}$ we find that the second quotient 2 is only the half of the first. The dividend being the same; the variation must arise from the divisor which is double; and it is evident that whenever the divisor increases, the same dividend cannot contain it as many times as before, and consequently the quotient which is nothing but that number of times, must be less. On the contrary, the quotient will be greater when the divisor becomes less, because that smaller divisor must necessarily be contained more times in the same dividend, as in the annexed division. $9)\frac{18}{2}$ and $3)\frac{18}{6}$. Hence,

59. 2dly, *The quotient* DECREASES *when the divisor* IN-

CREASES, *and on the contrary, it* INCREASES *when the divisor* DECREASES, or more concisely,

The quotient depends INVERSELY *on the* DIVISOR.*

We may then say that,

60. *The quotient depends* DIRECTLY *on the* DIVIDEND *and* INVERSELY *on the* DIVISOR.

61. Hence it follows that the quotient will remain the same, when the dividend and divisor are made the same number of times greater, that is, multiplied by the same number; for by multiplying the dividend, for instance, the quotient, which depends directly on it, will become a certain number of times greater; but, by multiplying the divisor, the quotient, which depends inversely on it, will become the same number of times less; therefore the increase, on one hand, being counterbalanced by the decrease, on the other, the quotient can experience no change, and must, consequently, remain the same. A similar result would take place should the dividend and divisor be divided by the same number.

2)8	Dividend and divisor, each 3 times greater	6)24
4	Same quotient,	4
6)48	Dividend and divisor, 3 times less,	2)16
8	Same quotient.	8

LONG DIVISION.

62. WHEN the divisor exceeds 9 or contains units, tens, hundreds, &c.*

1st Take for first dividual so many of the left hand figures of the dividend as will contain the divisor, ascertain the number of times they contain it; this gives the quotient: (place it at the right hand of the dividend).

2d Multiply the divisor by the quotient, and set down the product under the first dividual; units under units, tens under tens, &c. and then subtract it.

3d To the remainder affix the next figure of the dividend for second dividual. Try how often it contains the divisor, and set that second figure of the quotient on the right hand of the first.

4th Multiply the divisor by that second quotient, place the product under the second dividual, from which it must be

*See Appendix.

subtracted. To the remainder affix the next right hand figure of the dividend and proceed as before directed, until there remains no longer any figure of the dividend to affix to the last remainder.

63. REMARK.—If the *remainder*, increased by the next figure of the *dividend*, do not contain the *divisor* once, it contains it 0 times; then place 0 to the quotient and affix the next figure of the dividend to the same remainder. Should this be yet too small to contain the divisor, another 0 must be placed to the quotient, and the dividual be again augmented by the next figure of the dividend: continue in the same manner until you obtain a dividual large enough to contain the divisor.

64. The necessity of placing these ciphers in the quotient will easily appear, if we reflect that there must be on the right hand of the first quotient, as many *classes* as there are *figures* on the right hand of the first dividual: for, should that dividual be the only one, as it would give but one figure for the quotient, that figure must express units: if, besides the first dividual there was another figure, in the dividend, the quotient should then have two figures, that is, tens and units; and in short, each figure of the dividend must give one to the quotient, if not a significant one, it must be a cipher. Thus the left hand figure of the quotient will express units of the class it ought to represent.*

```
78)15836087(203026+59/78
   156
   ---
     236
     234
     ---
       208
       156
       ---
        527
        468
        ---
         59
```

Here seeing that 158 is greater than the divisor, I try how often it contains it, and finding it is 2, I set down 2 for quotient. Multiplying the divisor 78 by 2, and subtracting the product 156 from the first dividual, the remainder is 2. Now it is evident that since 78 is contained twice in 158, if these were units, the quotient would be units, if they were tens, that is, a quantity 10 times greater, the quotient would be tens, because it depends directly on the dividend; and since

* For, if we had 100 to divide by 50, as there is only one dividual, the quotient would be 2 units; but if we had 1000 to divide by 50, the dividend being 10 times greater the quotient must be 10 times greater also; that is, must express 2 *tens*, instead of 2 units. We must observe in this case, that there

158 expresses hundreds of thousands, the quotient must be hundreds of thousands, or represent units of the same class as the right hand figure of its dividual; consequently each of the figures on the right hand of that first dividual must furnish one for the quotient.

To the first remainder 2, I affix 3, the next figure of the dividend, and put a dot over it to mark which is the last class I make use of; and as the new dividual 23 does not contain 78, I put 0 in the quotient, and affixing 6 to 23, I have for new dividual 236, which contains the divisor 3 times, and leaves 2 for remainder. The addition of 0 to it, giving a number too small to contain 78, I find 0 for quotient and putting 8 with 20, I perceive that the divisor will be contained twice in the dividual 208, with a remainder of 52; affixing 7 to it, I find 6 for quotient: 6 is the last quotient figure, because I have no longer any figure in the dividend to affix to the rest 59.

65. Each new figure of the quotient must be placed on the right hand of the last so as to express units ten times less, because it is produced by a dividual 10 times less than the preceding one. In fact each partial quotient always expresses units of the same class as the right hand figure of the dividual which produces it.

66. If the dividend and divisor be terminated by zeros, an equal number may be suppressed in each, without producing any alteration in the quotient as already shown (article 61).

EXAMPLES.

400)17800	Div. 486000 by 9000(54	79480 by 80(993+4
———	21600 " 700(30+6	
44+2	14400 " 1200(12	

67. If the dividend has more 0's than the divisor, take from both as many as there are in the latter, and divide as usual.

EXAMPLE.

600)87000	Divide 126000 by 500	748000 by 8640
———	74800 " 140	36400 " 960
145	4870 " 290	

68. If the *divisor* has more 0's than the *dividend*, separate from this as many right hand figures as there are 0's in the

would be a second dividual; since all the dividend is not taken for the first, but only the 3 left hand figures. Hence we conclude, that the quotient always expresses units of the same class as the right hand figure of the dividual which produces it.

divisor, divide as usual, and affix the figures thus separated, to the last remainder for the true one.

EXAMPLE.

17|00)82|40(4
68
—
1440

I, here, first take off 40 from the dividend, and add it to 14, last remainder, to obtain the true remainder, 1440, which is smaller than 1700; or cutting off one 0 from these two last numbers, I have 144 to divide by 170 or $\frac{144}{170}$.

69. By division is found how many articles can be obtained for a certain sum when the price of one article is known. For instance:

If one yard of domestic muslin cost 12 cents, how many yards can I purchase for 180 cents?

It is evident that I can purchase as many times one yard as 12 cents, *the price of* 1 is contained in 180 cents, the *amount* I wish to spend. Hence I have to find how many times 12 is contained in 180, or divide 180 by 12, the quotient 15, indicates the number of yards.

70. By division is also found the value of *one* article when that of *several* is known. For instance:

If for 12 pounds of candles I gave 192 cents, how much was it per pound?

It is plain that if I had bought only 1 pound, the price would be 192 cents, that, if I had bought 2 pounds, each pound would have been only *one half* of 192, and consequently, to find the price of *each*, I must divide 192, the price of the *whole*, by 2, the number of articles, and so on if 3, 4, or 12 pounds. Therefore 192 divided by 12, gives 16 cents for quotient or answer.

71. A number is called *multiple* of another, when it contains the latter a certain number of times exactly, or without remainder, as 12 is the multiple of 2, 3, 4, 6; but not of 5—15 is a multiple of 3 and 5, but not of 2, 4, 6, &c.

72. A number is an *aliquot* part of another, when it is contained *exactly* in it. 2 and 3 are aliquot parts of 6—2, 3, 4, 6—aliquot parts of 12, but 5 and 7 are not. 2 and 5 are aliquot parts of 10; but 3, 4, &c. are not.

---

73. To prove multiplication, divide the product by one of its factors, the quotient will be the other factor, for since the product includes them *both*, when you divide by *one* of them, the quotient must necessarily express the other.

74. To prove division, multiply the quotient by the divisor,

to the product add the remainder if there be one, that sum will be equal to the dividend. This must evidently be the case, since the *dividend* containing the *divisor as many times as there are units in the quotient,* with a remainder; when that divisor is repeated as many times as there are units in the quotient, the product, augmented by the remainder must re-establish the dividend.

EXAMPLES. 3d *Set*

	Divide				Divide			remain.
1.	5,711,568	by	12	14.	44,916,210	by	527	
2.	5,711,568	"	24	15.	13,634,320	"	2008	
3.	269,352	"	36	16.	325	"	13	
4.	269,352	"	108	17.	4,509	"	36	(9)
5.	39,816	"	18	18.	266,328	"	243	
6.	39,816	"	36	19.	87,153	"	57	
7.	266,328	"	27	20.	600,006	"	500	(6)
8.	87,153	"	19	21.	60,800,900	"	8000	(900)
9.	600,000	"	50	22.	50,700	"	100	
10.	60,800,000	"	80	23.	3,024,005	"	4000	(5)
11.	50,700	"	10	24.	3,299,008	"	304	
12.	3,024,000	"	40	25.	44,916,216	"	1054	(6)
13.	3,299,068	"	608 (60 remain).	26.	13,634,407	"	4016	(87)

4th Set.

1.	Divide	178,464	by	16 (0 remain,)
2.	"	104,202	"	18 (0)
3.	"	266,328	"	27 (0)
4.	"	79,632	"	36 (0)
5.	"	937,386	"	54 (0)
6.	"	144,872	"	48 (8)
7.	"	1,575,360	"	72 (0)
8.	"	8,460,729	"	63 (18)
9.	"	547,129	"	87 (73)
10.	"	3,746,251	"	96 (43)
11.	"	487,564	"	83 (22)
12.	"	43,652	"	88 (54)
13.	"	244,556	"	71 (32)
14.	"	7,099,430	"	75 (5)
15.	"	3,510	"	15 (0)
16.	"	11,153	"	19 (0)
17.	"	19,895	"	23 (0)
18.	"	19,012	"	28 (0)
19.	"	942,340	"	35 (0)

20.	Divide	253,622	"	422	(0 remain)
21.	"	4,586,841	"	3467	(0)
22.	"	13,699,852	"	342	(16)
23.	"	1,345,680,000	"	120,000	(0)
24.	"	74,824,000	"	70,000	(64,000)
25.	"	943,246,042	"	86,449	(1003)
26.	"	74,297,224	"	7,890	(4984)
27.	"	1,680,000	"	36,000	(24000)
28.	"	46,447,760	"	1234	(0)

What is the fourth of twenty? How many fours in twenty?

Five being the fourth of twenty, what are the 2 fourths of twenty?—the 3 fourths of twenty?

What is the fifth of fifteen?—what are the 2 fifths of fifteen?—the 3 fifths?—the 4 fifths?

What are the 2 thirds of twelve?

If you receive 1 pistareen or 20 cents from your aunt, 5 dimes (or 10 cent pieces) from your uncle, half a dollar or 50 cents from your father, and 2 quarters of a dollar (or 25 cent pieces) from your mother, how much have you?

Ans. 170 cents.

If out of that money you spend 35 cents, and afterwards share the remainder equally among 3 of your friends, how much will each receive from you? 45.

What are the 3 sixths of 24?—the 4 sixths?—the 5 sixths?

One is what part of 3?—two is what part of 3?

One is what part of 5? three is what part of 5? 4 is what part of 5?

Two is what part of 8?—four is what part of 8?—6 is what part of 8?

What part of 10 is 2? is 4? is 6? is 8?

6 being the third of 18, what part of 18 is 12?

3 being the sixth of 18, what part of 18 is 6?—is 12?—is 9?—is 15?

4 being the fifth of 20, what part of 20 is 12? is 16? is 8?

What part of 24 is 3? is 9? is 15? is 12? is 18? is 21?

What part of a quarter of a dollar is 5 cents? is 10 cents? is 20 cents? is 15?

What part of a quarter of a dollar are $6\frac{1}{4}$ cents? are $12\frac{1}{2}$? are $18\frac{3}{4}$?

If you receive 20 cents, what part of 1 dollar (or 100 cents) is it?

Henry is to receive 3 fifths of 25 cents, how many cents

will he have? His sister Sarah is to receive the remainder, how many cents will she have? and what part of 25 will it be?

In 12 how many times 3? 4? 5? 6? 8? 7?

In 21 how many times 7? 4? 3? 5? 6? 8? 9? 2?

In 24 how many times 8? 6? 12? 3? 7? 4? 5? 9? 2? 10?

In 17 how many times 5? 6? 4? 2? 7? 3?

36 are how many times 6? 4? 12? 5? 9? 8? 3? 10? 2? 7? 11?

45 are how many times 7? 9? 4? 12? 5? 6? 8? 10? 3? 11? 2?

54 are how many times 3? 12? 9? 11? 4? 6? 5? 7?

66 are how many times 5? 11? 7? 6? 3? 2? 4? 9? 8? 12?

75 are how many times 6? 7? 9? 5? 8? 12? 10? 4? 11? 3?

99 are how many times 7? 9? 10? 8? 11? 12? 3? 6? 5?

120 are how many times 6? 10? 12? 3? 4? 9? 5? 2? 7? 8? 11?

Supplement to Multiplication.

75. To multiply by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. divide the other factor by 2, 3, 4, &c. and add the quotient to the product.

EXAMPLE.

What will 25 pounds of sugar cost at $12\frac{1}{2}$ cents per pound and at $18\frac{3}{4}$?

$$\begin{array}{rr} & 25 \\ & 12\frac{1}{2} \\ \hline & 300 \\ \frac{1}{2} \text{ of } 25 = & 12\frac{1}{2} \\ \hline & 312\frac{1}{2} \end{array} \qquad \begin{array}{rr} & 25 \\ & 18\frac{3}{4} \\ \hline & 200 \\ & 25 \\ \hline & 450 \\ \frac{1}{2} \text{ of } 25 = & 12\frac{1}{2} \\ \frac{1}{4} \text{ of } 25 = & 6\frac{1}{4} \\ \hline & 468\frac{3}{4} \\ \hline \end{array}$$

What cost 2 dozen oranges at $6\frac{1}{4}$ cents a piece? $1.50.

Do. 9 yards of cloth at $$5\frac{1}{2}$ a yard? $49.50.

Do. 16 barrels of flour at $$7\frac{3}{4}$ per barrel? 124.

In 36 pieces of cloth, each of $24\frac{1}{2}$ yards, how many yards? 882 yards.

The quotient of a certain number is $12\frac{1}{2}$, the divisor 8, what is the dividend? 100.

What number multiplied by 14 will make 112 ? 70 ? 56 ? 126 ?

What number multiplied by 7 will make 77 ? 56 ? 84 ? 784 ?

To find how many pounds Pennsylvania currency are contained in a given number of dollars, multiply the dollars by 3 and divide their product by 8. How many pounds Pennsylvania currency are there in 96 dollars ? £36.

To find how many dollars in a number of pounds Pennsylvania currency, multiply the pounds by 8, and divide their product by 3. How many dollars in £96 Pennsylvania currency ? $256.

DECIMAL FRACTIONS.

76. We have seen in the system of numeration, that the different *classes* of units *increase in a ten-fold ratio*, in going from the right to the left, and that they *decrease* at the same rate when going from the left to the right, until we reach the units, which is the least class.

We are now going to attend to the *division of the unit itself*, into parts, decreasing in a ten-fold ratio, and on that account called *decimals*.

77. To form a correct idea of what a *decimal fraction* is, conceive 1 unit divided into 10 equal parts, each part called a *tenth*: 1 *tenth* being divided into 10 equal parts, each part will then be the *tenth of a tenth*, or the *hundredth* of a unit. In the same manner, 1 hundredth being divided into 10 equal parts, each of them will, of course, be *the tenth of a hundredth*, or the *thousandth* part of a unit; and by continuing the same process, we shall have a succession of classes, *decreasing* by tens.

78. On the *left* of the units, the classes *increase* by a ten-fold ratio; on their *right*, they *decrease* at the same rate; the class of units becomes then the natural *barrier* between the *whole numbers*, and the *fractional parts*. Hence, a dot is placed on the right hand of the units. The whole numbers are then situated on the left of the dot, (called the *separatrix*) the decimals on the right.

millions	hundred thousands	ten thousands	thousands	hundreds	tens	UNITS	*tenths*	*hundredths*	*thousandths*	*tenths of thousandths*	*hundredths of thousandths*	*millionths, &c.*
6	7	4	2	3	1	5.	1	3	2	7	9	8
7th	6th	5th	4th	3d	2d	1st.	1st	2d	3d	4th	5th	6th
Integers.							Decimals.					

The figures on the *left* of the dot are *integers* or whole numbers; those on the *right* are *decimals.*

79. The 1st decimal class is invariably called *tenth;* the 2d, *hundredth;* the 3d, *thousandth;* the 4th, *tenth of thousandth;* the 5th, *hundredth of thousandth;* the 6th, *millionth,* &c. &c.

80. Thus if I wish to write down 5 tenths of thousandths; beginning to count on the right of the dot, I say, tenths, hundredths, thousandths, tenths of thousandths; the 5 belongs then to the 4th class of decimals, I set it down, and as there are neither tenths, hundredths nor thousandths given, I place 3 zeros between the dot and the 5, so as to make it express decimals of the 4th class, and the number is represented by 0.0005, if I choose to fill up the place of the units by a cipher, or else, .0005.

81. We write down a decimal number as a whole one, but we must particularly attend to the *name* of the figure last mentioned, and place it in its proper class; then if the figures on its left hand do not reach the class of the tenths, the vacancy must be filled up with as many ciphers as there are intermediate classes, and a dot put on the left.

If directed to write down seventy-eight hundredths: finding that the hundredths form the 2d decimal class, I put down 78 and make a dot on the left, so as to place the 8 in the second decimal column. If now I wanted to set down 178 hundredths of thousandths under the 78 hundredths; counting the classes, and finding that the hundredths of thousandths form the 5th class, I put 8 in that 5th class, 7 in the 4th, 1 in the 3d; and as the second and first classes are vacant, I put down 2 ciphers, and the dot on their left—thus: .78

.00178.

Place, one under the other, the following numbers:

4 units and 172 thousandths.—409 units and 6 tenths of thousandths.—0 units and 48 millionths.—2 thousands and 4 thousandths.—60 units and 6 tenths.—4 hundreds and 3045 tenths of millionths—and the decimal number ten thousands and sixty-four hundredths of millionths.

82. Should I be directed to write 64 tenths; observing that the last figure, 4, must alone be a decimal, I place the dot between 6 and 4; thus 6.4 and I have 6 units and 4 tenths.*

Write down 178 hundredths—2049 hundredths—six thousands—twenty-eight thousandths.

83. Read a decimal number as a whole number, only be careful to give to the last right hand figure, the name of the class it occupies, which may easily be found by counting from the tenth, until you reach said class. Thus, to read the number .0325; instead of saying separately 3 hundredths, 2 thousandths, and 5 tenths of thousandths, I at once say 325 tenths of thousandths, because 5, the right hand figure occupies the 4th class, or belongs to the tenths of thousandths.

EXAMPLES.

6.216—.104—6.028—20.49—.20547—.06—2046.2046—19.0019.

84. To make a decimal number 10, 100, &c. times *greater*, the dot must be moved 1, 2, &c. places to the *right*; for each figure will then, *by its local situation*, express quantities 10, 100, &c. times greater than before. For instance, if I wish to make 0.4806, ten times greater, I move the dot between the 4 and 8, thus: 4.806. In this last number, it is evident that the 4, which in the first number, expressed *tenths*, now stands for *units*; that is, for a quantity 10 times greater; and the 8 now expresses *tenths*, instead of hundredths, &c. &c.

85. To make a decimal number 10, 100, &c, times *less*, the dot must be moved 1, 2, &c. places to the *left*; for each figure will then represent quantities 10, 100, &c. times *less* than before. Thus: 7.46—to be made 100 times less, must be written .0746; for, the 7, which at first expressed 7 *units*,

* The subdivisions of dollars into 10 cent pieces, or tenths, and cents or hundredths, and mills or thousandths offer a complete specimen of decimal fractions and may serve to elucidate that system still more fully, if what we have said on the subject is not well understood.

Considering the dollar as the unit, its subdivisions become decimals, and they must be separated from the integer by the dot.

Write down 6 dollars and forty-two cents—4 mills—75 mills—905 mills—625 cents.

now expresses only 7 *hundredths*, or a quantity 100 times less than before; and it is the same as respects the 4 and 6.

86. Hence, to *multiply* or *divide* a decimal number by 10, 100, &c. it is sufficient to move the dot 1, 2, &c. places to the right or left.

Make 0.74 ten times less—0.74 ten times greater. 10.4607 one thousand times greater—0.764 one hundred times less—multiply 1.04, by 10—divide 1.04 by 10—divide 0.063 by 100—multiply it by 1000.

87. As long as the dot remains at the same place, each figure retains its local or relative value; of course, a *decimal number is not altered by placing* 1, 2 *or more ciphers on its right;* and in fact, it is evident, that 0.3, is the same as 0.30 or 0.300, &c. for in each of these expressions you have really but 3 tenths.

88. But the result would be very different, if we were to place the ciphers between the dot and the 3; as, 0.03—0.003—for, in these cases, as already noticed, (article 84) by the removal of the dot, we alter the *relative* or *local* value of the 3, which, by the first alteration, becomes 3 hundredths, and by the second 3 thousandths, instead of 3 tenths.

ADDITION OF DECIMALS.

89. From the formation of decimals, it is evident that they must be added just like whole numbers. Particular care however, must be taken in placing the same classes under each other; and from the right hand of the amount, you must mark off, with a dot, as many decimals, as there are *decimal classes* in the number which contains the most.

EXAMPLE.

3.056	After placing the same classes under each other,
.0746	I sum up, as in common addition, beginning at the
1.23	right hand column, and from the right hand of the
4.00695	amount, 836755, I separate 5 decimals, because the
———	number which contains the most has 5.
8.36755	
———	
0.11100 Proof.	

Add together 6.23—0.0058—47.—6.25003—7.007—0.0085.

Add 7 thousandths—6 units and 42 millionths—9 hundredths—6 tenths—45 units—two hundred and sixty-seven

hundredths of thousandths—forty-five thousands, six hundred and fifty-four millionths. Ans. 51.745366

SUBTRACTION OF DECIMALS.

90. THE SUBTRACTION of decimals is made exactly as the subtraction of whole numbers. You borrow in the same manner, and, for the same reason, you may convert the borrowed unit into 10 of the next inferior order.

91. The only case which offers any difficulty, and which *seems* to present an insurmountable obstacle, is, when the *subtrahend* or lower number, has more decimal figures than the superior or upper; for, in such a case, the student knows not what to subtract from. For instance: when directed to ascertain the difference between 6.045
and 4.07486.
he knows not what the two right hand figures 6 and 8 are to be subtracted from. In all similar cases let him make the number of decimal figures *equal* in both numbers, by placing at the right of that which has the least, as many ciphers as will make up the number of classes. So, in the above, after adding 2 zeros on the right of the 5 thousandths, he may subtract as usual, and find 1.97014 for difference, after pointing off as many decimals, as there are in the number which contains the most.

The addition of those ciphers on the right is permitted, since, as we have already seen (art. 87) they do not alter the value of the decimal number.

EXAMPLES.

7.004	0.6486742	32.105
0.64867	0.07004	30.999962

Subtract 0.6204 from 5 units—72 thousandths from 1 tenth—67 hundredths from 2 units and 4035 tenths of thousandths—0.444 from 1 unit and 43 millionths.

MULTIPLICATION OF DECIMALS.

92. MULTIPLY decimals as if they were whole numbers, and by a dot, separate, on the right hand of the product, as many figures as there are decimals in *both* factors.

93. For, by considering the decimals in the factors as *integers*, you make these factors 10, 100, &c. times *greater*

than they really are; the product, which depends directly on the value of its factors, (art. 42), will then be 10, 100, &c. times greater than it ought to be; it must, of course, be brought back to its true value; and that is done by pointing off, with a dot, as many figures as there are decimal classes in both factors.

EXAMPLE.

```
  6.042
  1.298
 ------
  48336
 54378
72504
--------
7.842516
```

By multiplying as if both factors were integers, I consider the dot as removed, and of the 2 thousandths of the multiplicand, I make 2 units, that is, a quantity one thousand times greater than it really is; the product, which *increases* as the multiplicand does, will then be 1000 times *greater* than it ought to be; it is then wrong; to rectify it, we must try to make it 1000 times *less*; and that is effected by cutting off 516, or the 3 right hand figures of the product, or, as many figures as there are decimals in the multiplicand.

The *multiplier* being likewise made 1000 times greater than it ought to be, by the removal of the dot, (for the 8 is then units instead of thousandths) will give a product 1000 times too great, since the product depends on the multiplier: to bring it to its true value, we must point off 3 more figures that is, 842.—The dot is then between the 7 and 8; the product containing 6 decimal classes, that is, as many as there are in both factors.

94. We might say that the *multiplicand* being 1000 times too great, and the *multiplier* 1000 times too great also, their product will be 1000 times 1000, or 1,000,000 times too great; to reduce it to its exact value, it must be made 1,000,000 times less; and that is done by separating 6 figures by a dot; for the 7 now expresses 7 units, instead of 7 millions.

95. If the product do not contain as many figures as there are decimals in both factors, add on its left, as many zeros as will complete the number of classes, and prefix a dot.

EXAMPLE.

```
1. 1.0024
   0.00027
   -------
     70168
    20048
   -------
    270648
```

Now as I have only 6 figures in the product, and as there must be 9 decimals in it; to the *left* hand of the product, I add 3 ciphers, and prefixing a dot, I find 0.000,270,648, for the real product.

EXAMPLES.

2.	Multiply	0.362	by	206.
3.	"	5789.	"	0.018
4.	"	0.3467	"	1323.
5.	"	13.0548	"	2.911 and to the product add

38772 tenths of millionths.

6.	Multiply	16.	by	2.44
7.	"	101.	"	0.09
8.	"	3.71	"	1.26 and to the product add

54 tenths of thousandths.

9\. Multiply 2.1162 by 14. and add to the product 0.844

10.	Multiply	0.1791	by	0.268 (add 0.000012)
11.	"	1.55	"	0.480 (.026)
12.	"	28.08	"	0.34
13.	"	8.47	"	10.
14.	"	20.102	"	0.39 (add .022)
15.	"	29.831	"	0.952
16.	"	865.	"	0.23
17.	"	38.5	"	0.5
18.	"	3.85	"	0.5
19.	"	500.	"	0.385
20.	"	9.	"	1.
21.	"	10.	"	0.9
22.	"	100.	"	0.09
23.	"	0.01	"	9.
24.	"	9.	"	0.001
25.	"	000.285	"	0.0003
26.	"	4.001	"	0.0004
27.	"	99.31	"	79. (add 0.61)
28.	"	246.	"	40.6504 (add .0016)
29.	"	88.	"	7.011 (add .032)
30.	"	865.1304	"	23. (add .0008)
31.	"	2212.	"	36. (add 10.)
32.	"	0.007	"	0.007
33.	"	0.006	"	0.012

DIVISION OF DECIMALS.

96\. The division of decimals is made in the same manner as the division of whole numbers. It is, however, necessary to prepare the dividend and divisor, before beginning the division.

97. If the dividend and divisor have each the same number of decimals, remove the dots and divide as in whole numbers; for by that removal you render them both the same number of times greater, or, in other words, you *multiply* them by the same number; and we know that the *quotient* suffers no alteration when the dividend and divisor are *both* multiplied by the same number (art. 61).

EXAMPLES.

1. Divide 78.12 by 1.24—as there are 2 decimals in each, I suppose the dot removed, and have then 7812. to divide by 124.

2. Divide	74.572	by	0.362	5. Divide	38.0064	by	13.0548.
3.	"	104.202	"	0.018	6.	"	39.04	"	2.44
4.	"	458.6841	"	0.3467	7.	"	9.09	"	0.09

98. If the dividend and divisor have *not* an equal number of decimals, add, on the right hand of that which has the least, as many zeros as are wanted to make the number of classes the same in both; this we know (art. 87) can be done without altering the value of the decimal number. When brought to that state, suppose the dots removed, and divide as in whole numbers. Ex. 1. Divide 390.4 by 0.244. Observing that the divisor has more decimals than the dividend, I add 2 ciphers to the latter, and supposing the dots removed, I have 390400, to divide by 244, the quotient of which, by common division, is 1600 units.

99. It may happen that after having made the number of decimals equal in the dividend and divisor, the latter is not contained in the former; this shows that the quotient will contain no integers; therefore, put down 0, in the quotient, to represent the class of units and placing a dot on its right, proceed as follows:

Put, on the right of the dividend, as many *ciphers* as you wish to have *decimals* in the quotient, and when the division is performed, the *last* quotient figure must be of the class indicated by the number of ciphers added to the dividend; that is, the 3d class if 3 zeros have been added; the 4th, if 4; the 6th, if 6; &c.

Divide .02688 by 4.80

```
           ..
480000.)2688.0000(0.56
        2400 000   0.0056
        --------
         288 0000
         288 0000
        --------
               0.
```

As there are five decimals in the dividend, and only 2 in the divisor, I add 3 to the latter, and removing the dots, I have to divide the whole number, 2688 by 480,000. But as the dividend does not contain the divisor; I conclude that there is no integer in the quotient;

then put 0 with a dot, to fill up the place of the units. Wishing to have 4 *decimals*, I put 4 *ciphers*, and dividing, find 56 for quotient. But, as by adding 4 ciphers to the *dividend* I made it 10,000 times greater, the *quotient*, which depends directly on the dividend (art. 57.) is 10,000 times too great; of course, to counterbalance this increase, I must make that quotient express tenths of thousandths, instead of units: I have then 0.0056.

100. The same operation may be explained as follows:

After the dividend and divisor are both prepared, I reduce the *dividend* into *tenths*, by adding 0 to it. The quotient resulting from it must then express *tenths*, and as the divisor is contained no times in that dividual, I put 0 tenths in the quotient. Adding another 0 to the last dividual, it becomes hundredths; the quotient will then express hundredths; as I cannot yet divide, or as the divisor is contained 0 times in the dividend, I put 0 in the class of hundredths, of the quotient. Adding now another 0 to the last dividual, I find it contains the divisor 5 times; I then set down 5 in the class of thousandths, of the quotient; to the remainder, adding another 0, I find 6 tenths of thousandths; and I have finally as many classes of decimals in the quotient, as I added zeros to the dividend and dividuals.

```
480000)2688.000 (0.0056
       2400 000
       --------
        288 0000
        288 0000
        --------
             ..0
```

101. When the dividend and divisor are both prepared and brought to express whole numbers, we may, instead of adding ciphers to the dividend, successively cut off those which are in the divisor; the last quotient figure, must, as in the other case, be of the class indicated by the number of noughts cut off.*

102. When there is no longer any noughts to cut off in the divisor, if you wish to have more decimals in the quotient,

* The reason of this process is simply this: by cutting off 1, 2, &c. noughts from the divisor, you make it 10, 100, &c. times smaller; the *quotient*, depending *inversely* on the *divisor*, (art. 59) will of course be 10, 100, &c. too great; to reduce it to its true value, you must make it express quantities 10, 100, &c. times less than units, that is, decimals.

add noughts to the successive remainders, until you have as many classes as you want.

1st example 4.80)0.02688	As the dividend does not contain the divisor, the quotient will
prepared 480000(2688	express 0 units. Now instead of
48~~0000~~)2688(0.0056	adding 0 to the dividend, I cut off one from the divisor, and of course
2400	make it 10 times less; the quotient,
288	if considered as units, would be
288	10 times too great, I have then to make it express tenths, and as the
0.	divisor is not contained once in the dividend, I put 0 tenths in the

quotient, and, finding that after cutting another 0 from the divisor, the remaining figures 4800 are not contained in the dividend 2688, I put 0 in the quotient; that 0 expresses hundredths in order to compensate for the diminution of the divisor. Cutting off a third 0 from the divisor, I find that 480 is contained 5 times in the dividend, and that 288 remain. Cutting off the fourth 0 of the divisor, I find that 48 is contained exactly 6 times in 288, so that I obtain the same quotient as before, viz. 0.0056.

Divide 0.2764 by 4.6, with 6 decimals in the quotient.

4.6)0.2764—prepared, it becomes 4.6000)0.2764—removing the dots to have whole numbers, we obtain,

46~~000~~)2764(0.060086	which give no units. I then cut off the
2760	right hand 0 of the divisor, and as it is not yet contained in the dividend, I put
400	0 tenths in the quotient. Cutting off
368	another 0 in the divisor, I find 6 hundredths for quotient, and 4 is the re-
320	mainder. As, after cutting off the last
276	cipher of the divisor, it is not contained in the dividual 4, I put down 0 thou-
44	sandths to the quotient; and now, as I

have no more ciphers to take from the divisor, I add one 0 to the dividual 4, so as to have 40; but as this does not yet contain the divisor 46, I set down 0 in the quotient, and adding another 0 to the last dividual 40, I am enabled to go on with the operation and find .060086 for quotient, and 44 for remainder.

Divide				Divide			
8. "	4.68	by	1.26	18. "	1.925	"	3.85
9. "	30.4708	"	2.1162	19. "	192.5	"	0.385
10. "	0.048	"	0.268	20. "	9.	"	9.
11. "	0.746	"	0.48	21. "	9.	"	0.9
12. "	9.5472	"	0.34	22. "	9.	"	0.09
13. "	84.7	"	8.47	23. "	0.9	"	9.
14. "	7.84	"	0.39	24. "	0.09	"	9.
15. "	28.399112	"	29.831	25. "	0.000855	"	0.285
16. "	198.95	"	865.	26. "	0.016004	"	4.001
17. "	19.25	"	38.5				

103. When, in the division of whole numbers, you have found the units of the quotient, and have a remainder, you may, if you wish to have the quotient with more exactness, add at once, or successively, as many *ciphers* as you wish *decimal classes*. This is evident from what has already been said.

In fact, the remainder of a division, *being always less than the divisor*, may be compared to the dividend prepared, (art. 99) and of course, that remainder may be treated exactly in the same manner.

27.	Divide	7846	by	79	with	2	decimals.
28.	"	10000	"	246	"	4	"
29.	"	617	"	88	"	3	"
30.	"	19898	"	23	"	4	"
31.	"	79642	"	2212	"	2	"

PROMISCUOUS EXAMPLES.

1st Add together seventy units, and forty-four thousandths —2 units, and 78 hundredths—48 thousands, and 48 thousandths—twelve hundred and fifty-nine units, and 6 tenths of thousandths—multiply the sum of those numbers by 7 units and 4 hundredths.—N. B. Let the pupil point off the decimals 347297383104.

A man has 2 acres and seventy-six hundredths of land; another has 2 acres, and 82 thousandths; who has the most, and what is the difference?

Difference, six hundred and seventy-eight thousandths.

A farmer has 7 lots of land, each containing 8 acres, and two hundred and forty-nine thousandths: has he more land than another farmer who has 8 lots each containing 7 acres, and two hundred and forty-nine thousandths?—If they have not the same quantity, what is the difference? Diff. 0.249

A man is to have $\frac{1}{3}$ of the quantity of hay he mows; there

are 12 tons and 64 hundredths of a ton of the hay. What will be his portion? If, out of this portion, he gives 1 ton and 23 hundredths to a man who helped him to mow, and sells the half of the remainder; how much hay will he have left?—(point off the decimals) 14917

A garden is 28 yards, and 63 hundredths long, and 14 yards eight hundredths broad: if I run a fence equal to the length of the garden, so as to cut off a breadth of 2 yards 7 tenths; how much ground will remain in the garden, or how many square yards* will it contain?

yds. 3258094 point off the decimals.

If when the $6\frac{1}{4}$ cents loaf of bread must by law, weigh 15 ounces 6 tenths; a baker puts in his loaf only 15 ounces and 3 tenths; what does he gain on each loaf; and on what number of loaves will he gain one? 1 on 51 loaves.

A lady has a parlour 17 feet 45 hundredths long, and 15 feet 8 hundredths broad; she wishes to cover it with a patent floor cloth: the maker charges by the square foot; for how many feet must he charge? Point off the decimals 2631460.

To 9 units, and nine hundred and ninety-nine thousands, nine hundred and ninety-nine millionths, add 1 millionth; what sum will you have? 10.

GEOMETRICAL RATIOS.†

104. When we compare two numbers, in order to find how many times the one contains the other, the quotient is called the *geometrical ratio*: thus, 6 containing 2, three times; 3, is the geometrical ratio.—8 being contained 7 times in 56; 7, is the geometrical ratio of 8 and 56.

105. The two quantities compared are called the *terms* of the ratio. They are written down one after another and separated by this mark : called a colon: as 4 : 8, read 4 is to 8.

106. The 1st term is called *antecedent*; the 2d, *consequent*. When the antecedent is *less* than its consequent, the ratio is

*Note.—To find the number of square yards, feet, &c. of any object; multiply the *length* by the BREADTH—so, a table 4 feet 28 hundredths long, and 3 feet 7 tenths broad, contains 15 feet, 846 thousandths; the product of 4. 28 the length, by 3. 7 the breadth.

† Besides *geometrical* ratios, there are *arithmetical* ratios, which indicate the difference of two numbers; but as they are not of any immediate use, we postpone their consideration; and will do the same as respects arithmetical proportion.

ascending. When the antecedent is *greater* than its consequent, the ratio is *descending*.

In 12 : 4 which is the antecedent? which the consequent?

In 8 : 16 which is the antecedent? which the consequent?

Is 4 : 12 an ascending, or a descending ratio? and why?

Is 12 : 4 do. do. do.

Do. of 6 : 9?—14 : 2?—8 : 72?—84 : 12?—9 : 15?—15 : 9?—12 : 40?

107. Since the geometrical ratio is nothing but the quotient of a division, and since, both the dividend and the divisor of any division, may be multiplied or divided by the same number without changing the value of their quotient, (art. 61) it follows, that,

108. *Both terms of a geometrical ratio may be multiplied or divided by the same number without altering that ratio.* The ratio of 8 : 4, is 2; if we multiply both terms by 3, we have the new terms 24 : 12; for $8\times3=24$, and $4\times3=12$, which give 2 for ratio. If 6 : 30, be divided by 6, the new terms 1 : 5, evidently give the same ratio, $\frac{1}{5}$; obtained by dividing the first term 1, by the second 5, expressed by $\frac{1}{5}$—or 6 by 30; $\frac{6}{30}=\frac{1}{5}$; or by dividing the second term 5 by 1, or 30 by 6; the quotients of which are in both cases 5.

109. To find the ratio, some divide the *antecedent* by its *consequent*; others, on the contrary, divide the consequent by its antecedent. It is perhaps as well to divide the greater term by the less. Ex.—7 : 42.—To find the ratio, I divide 42, the greater number by 7, the less; and the quotient 6, gives me the ratio.

Find the ratio of 4 : 12—9 : 27—70 : 5—24 : 6—30 : 5—3 : 7—5 : 9—5 : 30—6 : 24.

110. Of the two equal ratios 6 : 30 and 1 : 5; the second is obviously presented under a simpler form, or reduced to its lowest terms; and as it was done by dividing the two terms 6 and 30 by the same number 6, it follows that:

111. *Any geometrical ratio may, without altering its value, be simplified or reduced to its lowest terms, by division.* Simplify 6 : 42—$\frac{3}{15}$—$\frac{4}{10}$—50 : 60—$\frac{8}{36}$; 7 : 21—15 : 36—42 : 63—540 : 810—4 : 12—14 : 70—16 : 24—9 : 12—68 : 76—5 : 44—64 : 128—105 : 185—23 : 37.

112. The greater term is always equal to the less, multiplied by the ratio. The less term is, of course, equal to the greater divided by the ratio. Ex. In 12 : 84; the greater term 84 is equal to the less 12 multiplied by the ratio 7, as $12\times7=84$; and likewise 84 divided by 7 equals the less term

12—so when we know the first term of an ascending ratio and the ratio, we obtain the second term by multiplying the first by the ratio. If in an ascending ratio the antecedent is 9, and the ratio 4, what is the consequent?—$9\times4=36$.—Antecedent 7, ratio 9, find consequent. Antecedent 2, ratio 5; what is the consequent?—The consequent 36 the ratio 4, find the antecedent—the consequent 18, ratio 3, what is the antecedent? In a descending ratio the antecedent being 6, the ratio 3, what is consequent?—antecedent 21, ratio 7, what is consequent?—The consequent 9, ratio 3, what is antecedent?—the consequent 72, ratio 6, what is antecedent?—Antecedent 48, ratio 3, what is the consequent?

GEOMETRICAL PROPORTIONS.

113. A GEOMETRICAL PROPORTION is the connection of 2 equal *geometrical* ratios, both ascending or descending, as: 8 : 16 : : 6 : 12 or 6 : 3 : : 8 : 4; read as follows: 8 is to 16 as 6 is to 12—6 is to 3 as 8 is to 4. The first, means that 8 is the same portion of 16 as 6 is of 12; the second, that 6 contains 3 as many times as 8 contains 4; which is evidently true. Hence, the *combination* of the above ratios forms *geometrical proportions*. But, it would not be so if we had 2 : 6 : : 4 : 8; because 3, the geometrical ratio of 2 to 6 is not the same as 2, the ratio of 4 to 8.*

6 : 24 : : 2 : x=to what number?
12 : 4 : : 9 : x.
4 : 8 : : 7 : 21, is this a proportion?
5 : 20 : : 1 : x=to what number?
12 : 6 : : 9 : 3—is that right?
16 : x : : 24 : 6—

114. The *first* and *last* terms of a proportion are called the EXTREMES: (probably from their situation at each extremity of the proportion) 8 and 12 are the *extremes* of the first proportion above. What are the extremes of the others.

115. The *second* and *third* terms are called MEANS; 16 and

* A geometrical proportion may also be defined:
1st. A succession of 4 numbers of which the first contains the second as often as the third contains the fourth, as: 6 : 2 : : 9 : 3, in which it is plain that the first quantity 6, contains 2, three times; that is, as often as the third 9, contains the fourth 3.
2dly. A succession of 4 numbers, the first of which is contained in the second as often as the third is in the fourth—9 : 3 : : 6 : 2—8 : 4 : : 18 : 9.

6 are the *means* of the first proportion. Name the means of the others.

116. Since in a ratio, the two terms are called *antecedent* and *consequent*; a proportion, having 2 ratios, must, of course, have 2 antecedents and 2 consequents, viz. the *first* antecedent and its consequent; the *second* antecedent and its consequent.

In 6 : 9 : : 4 : 6 which are the antecedents? which the consequents? What is the consequent of the first antecedent? What is the antecedent of the second consequent? In 8 : 4 : : 32 : 6, does the first consequent contain its antecedent? and does the second antecedent contain its consequent?

117. The two ratios must, as already said, be not only equal, but both *ascending* or both *descending*. Consequently 12 : 3 : : 8 : 32 do not form a proportion; for, altho' the ratio is 4, in both cases, yet the first ratio is a descending one, while the second is ascending.—3 : 21 : : 63 : 9, do not form a proportion; for, the first ratio is ascending, and the other descending.

Place the above numbers so as to form proportions, and do the same with the following ones.

7 : 49 : : 70 : 10	7 : 63 : : 108 : 12
8 : 4 : : 7 : 14	24 : 312 : : 65 : 5
11 : 121 : : 55 : 5	136 : 8 : : 4 : 68

Why do they not form proportions as they are?

The necessity of a proper arrangement of the terms of a proportion, should be well understood, being of the first importance in stating the rule of three.

4 : 6 are two terms of a proportion, the two others are 12 and 18, in what order must they be placed? 8 and 24 are the 3d and 4th terms of a proportion, the other terms are 63 and 21, how must they be placed?

9 : 6 are the 3d and 4th terms; 8 and 12 the others: form the proportion.

7 : 35 are two terms; the others are 65 and 13 : form the proportion.

x, the 1st term of a proportion is not known exactly, we know it, however, to be greater than the 2d term 14: how must 17 and 102 be placed so as to form a proportion with x : 14?

15 : 75 are the 3d and 4th terms of a proportion: place 145 and 29 so as to complete it.

x, the 4th term, is greater than 70, the 3d: place 48 and 8, so as to form a proportion.

x, the 1st term, is less than 66: place 183 and 23, so as to form a proportion.

6 and 12 are the 2d and 3d terms: place 8 and 9 so as to form a proportion.

x, the 4th term, is less than 204: complete the proportion with 27 and 422.

118. *In every proportion the product of the extremes is equal to that of the means.*

This is a fundamental and very useful principle of proportions.

$6 : 12 : : 4 : 8$ the product of the extremes $8 \times 6 =$ evidently 12×4, the means.

$7 : 21 : : 6 : 18$; 21×6 means $= 126$ and 7×18 extremes $= 126$.

$18 : 9 : : 4 : 2$; $18 \times 2 = 9 \times 4$.

But it is not so of $18 : 9 : : 6 : 4$ in which $18 \times 4 = 72$ and $9 \times 6 = 54$; therefore, those 4 numbers are not proportional.

This fundamental principle may be explained as follows: Suppose we have the proportion $6 : 2 : : 9 : 3$—by multiplying the two consequents, that is, one means and one extreme, by the ratio 3, we obtain $6 : 6 : : 9 : 9$. In this last proportion, it is evident that the product of the extremes 6 and 9 is equal to that of the means 6 and 9, since they both result from the two same factors 6 and 9. But since they are equal now, they must have been equal before, for had they been unequal they would still remain so, after being multiplied by the same number 3, or, in other words, after being equally increased. Hence we conclude they were equal.*

119. This fundamental principle enables us to find any one term of a proportion when the 3 others are given. To find the first term or first extreme, for instance, multiply the two means together, and divide their product by the known extreme.† Find the 1st term of $x : 42 : : 18 : 126$.

$42 \times 18 = 756$ product of the means: dividing it by the

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* The same thing may be demonstrated as follows—If the 1st *antecedent*, which is always an extreme, be greater or less than its *consequent* or 1st means, by the quantity indicated by the ratio, in compensation, the 2d antecedent or 2d means, will be the same number of times greater or less than its consequent or 2d extreme. So that the increase in one product is counterbalanced by an equal increase in the other product, and they, of course, will be equal.

Let the teacher select the proof that appears to him the best, if he do not choose to explain both.

† See appendix.

known extreme 126, the quotient 6 gives the other extreme. So $x=6$ and the proportion is 6 : 42 : : 18 : 126. Find

the 1st terms of	x : 78 : : 14 : 6	x : 24 : : 192 : 96
	x : 12 : : 117 : 9	x : 6 : : 7 : 1
	x : 468 : : 16 : 144	x : 126 : : 14 : 49

120. To explain this operation, we must recollect that it has been proved (art. 73) that, *if we divide a product by one of its factors, the quotient will be the other factor.* This premised: when, by the multiplication of the 2 means, we have the *product* of the means, it is just as if we had that of the extremes; (since the product of the means is equal to that of the extremes) consequently, knowing the product of the extremes and one of its factors, viz. the known extreme, we can easily find the other, by dividing by the one we know.

x : 12 : : 8 : 6—The product of the means $12\times8=96$. But since the product of the extremes is equal to it, I conclude it is also 96; and this 96 must arise from the multiplication of the *known* extreme 6, by another *unknown* number. How am I to find that other number? Simply by dividing the product 96, by its known factor 6; the quotient 16, gives me the other factor, or other extreme: so that the proportion becomes 16 : 12 : : 8 : 6—in which $16\times6=96$ and $12\times8=96$.

121. Having shown (art. 108) that geometrical ratios may be simplified by dividing both terms by the same number; it follows, that a *proportion*, being composed of 2 ratios, may be simplified in the same manner: thus:

1st—6 : 78 : : 14 x: becomes 1 : 13 : : 14 : x, by dividing the first ratio, or 1st antecedent and its consequent, by 6, which is contained exactly in both; in this, the 4th term is found by merely multiplying 13 by $14=182$.

2d—x : 12 : : 117 : 9—becomes, by dividing the 2d ratio by 9—x : 12 : : 13 : 1.

3d—x : 192 : : 24 : 96—becomes, by dividing the 2d ratio by 24—x : 192 : : 1 : 4.

In these two simplified proportions, the first term is more easily found, since you have only, in the first, to multiply 14 by 13, without having any division to perform; for it is useless to divide by 1; and, in the second, the extreme is found by merely dividing 192 by 4: for there is no occasion to multiply by 1.

122. Some practice is necessary to make these simplifications expeditiously. The following directions will facilitate that operation. As both terms must always be divided without a remainder; when the numbers are both even, try if they

can be exactly divided by 8, 6 or 4, and divide accordingly: if it cannot be done, they must be divisible by 2. When one of them is odd, or both of them, try to divide by 3, 5, 7 or 9. If you cannot readily do it, it is better at once to multiply and divide the given numbers as they are, than to waste time in trials which may prove fruitless. When both terms are terminated by *zeros*, we know that the same number may be cut off from each of them.

As respects 3 and 9; sum up the figure representing the numbers; if the amount contains 3 exactly, the number is divisible by 3—if it contains 9 exactly, you may divide by 9. Ex. 1131—I say 1 and 1 equal 2 and 3=5 and 1=6, and as 6 contains 3 exactly, I conclude that 1131 is divisible by 3; and, in fact, the quotient is 377, without a remainder. This last number 377, cannot be exactly divided by 3; for the sum of its figures is 3+7+7=17, a number which is not a multiple of 3.

4653—is divisible by 9, because the sum of its figures 4+6+5+3=18 which contains 9 exactly. Also, after dividing I have 517, without remainder.

$x : 15 :: 336 : 840$.—To find the value of x, I would multiply 336 by 15, and divide the product by 840.

$$\begin{array}{r} 336 \\ 15 \\ \hline 1680 \\ 336 \\ \hline 84\cancel{0})504\cancel{0}(6 \\ 504 \\ \hline 0 \end{array}$$

Let us see what these operations will be, after simplification. The 2 numbers, 336 and 840 being both even numbers, are divisible, either by 8, 6, or 4. I immediately see that 8 is a common factor, and dividing by it, I find $x : 15 :: 42 : 105$—as one of these, viz. 105 is an odd number, while 42 is even, I cannot divide them both by an even number; then I will try to do it by 3,—and finding that the sums of their figures 4+2=6 and 5+1=6 are multiples of 3 I conclude that the division by 3 may be performed; and I obtain $x : 15 :: 14 : 35$—I at once see, that 14 and 35 are both divisible by 7, so as to give $x : 15 :: 2 : 5$; and in this proportion, I perceive at once that x is equal to 6.

These simplifications are extremely useful, and the student should be well exercised on them.

EXAMPLES.

$x : 9 :: 42 : 14$	$42 : 72 :: 8 : x$
$16 : 72 :: 3 : x$	$x : 68 :: 12 : 51$
$x : 5 :: 100 : 16$	$x : 28 :: 18 : 63$

123. Since the product of the means is equal to that of the extremes; and since products *decrease* as their factors; (art. 42) it follows, that when we divide the 1st extreme, for instance, by any number, if, at the same time, we divide *either of the means by the same divisor;* the equality of the products will not be destroyed. Hence we may *simplify* any proportion by dividing *one extreme*, and *either of the means* by the same number; or one *means* and either of the *extremes*.

EXAMPLE.

$x : 9 :: 14 : 21$—After dividing 14 and 21 by 7, the proportion becomes,

$x : 9 :: 2 : 3$—Now, observing that 2 and 3 are not divisible by the same number; but that 3, one extreme, and 9, one means, are both divisible by 3, I divide, and find $x : 3 :: 2 : 1$—which at once gives 6 for the value of x.

I have not altered the equality of the products; for, if I made the product of the extremes 3 times *less*, by dividing; I counterbalanced that diminution, by making, at the same time, 9, one of the factors of the means 3 times less. Simply the following proportions.

$7 : 35 :: 13 : 65$	$x : 48 :: 78 : 27$	$84 : 108 :: x : 87.$
$210 : 315 :: 46 : 69$	$x : 84 :: 87 : 108$	
$28 : 105 :: 16 : x$	$9 : x :: 36 : 78$	

124. In any geometrical proportion, we may add each *consequent* to its *antecedent*, and comparing the *sums* with the *consequents*, form a new proportion: for, each antecedent being *increased* by its consequent, will contain it once more than before; so that each ratio, increasing by 1, they will yet remain equal.

Ex. $24 : 8 :: 15 : 5$—becomes by adding each consequent to its antecedent $24+8 : 8 :: 15+5 : 5$ equal to $32 : 8 :: 20 : 5$, in which the two ratios are 4, instead of being 3, as in the first one.

The sum might likewise be compared to the antecedents.

EXAMPLE.

$6 : 18 :: 9 : 27$ becomes,

$6+18 : 6 :: 9+27$ is 9 equal to $24 : 6 :: 36 : 9$.

THE RULE OF THREE.

We are now going to direct our attention to the *rule of* [three], which, from its importance and usefulness, is sometimes

called the GOLDEN RULE. It is, by some, divided into the *single* rule of three DIRECT and INVERSE; and into the *compound* rule of three *direct* and *inverse:* but as by the following method of stating it, the distinction between *direct* and *inverse* becomes useless; and as that distinction, wherever introduced, merely tends to perplex and vex students, we gladly dispense with it; and will treat only of the *single rule of three*, and the *compound rule of three.*

THE SINGLE RULE OF THREE,

125. TEACHES how to solve every question in which, three terms of a proportion being given, the fourth is required.

As we know already how to find any term of a proportion when the others are set down; it remains only to give the directions necessary to state the question. A few preliminary observations must be made.

126. In questions solvable by this rule, *the terms are always given by pairs;* or two terms of the same name, or kind, or nature; or, perhaps more correctly, two *homogeneous* terms. There is always a term of the same name or kind as the required answer, or *homogeneous,* with it; as in the following problems.

1st. What is the value of 15 yards muslin, at the rate of 56 cents for 3 yards?

2d. How many men will in 16 days do what 28 men did in 12 days?

3d. How many pounds of sugar can I buy for $7.50 at the rate of 75 cents for 5 pounds?

4th. How many yards of carpeting, 1 yard wide, must I buy to cover a floor 10 yards broad, and 18 yards long?

5th. If a principal of 120 dollars, bring $7.20 interest, how much interest will a principal of $600 bring?

1st. In the first example, we want to know the value of some muslin; that value must be expressed in dollars and cents; and we find in the question the term 56, which expresses cents, a term which bears the same name as the answer, or is homogeneous with it: those 2 terms form a pair. 3 yards, and 15 yards are the other homogeneous terms, or the other pair.

2d ex. Here, *men* are wanted: one number of men is given; of course, we have one pair of terms. The 2 numbers of *days*, form the other pair.

3d ex. 5 pounds of sugar given; and, a number of pounds required: that is one pair. $7.50 and 75 cents, being prices, are homogeneous, and consequently form the 2d pair.

4th ex. Here, every term expresses yards. We shall, however, find them matched or paired, if we observe that the floor has two dimensions, viz. length and breadth; and that only one of the dimensions of the carpeting, viz. its breadth, is given. Hence 18 yards, the length of the floor, and the unknown length of the carpeting form one pair of terms, and the other pair consists of 1 yard and 10 yards, the two given breadths, or 2 homogeneous terms.

5th ex. Every term is expressed in dollars, or dollars and cents; but we must take notice that although they are all of the same name, those terms are not properly *homogeneous;* for, some represent *capitals* or *principals*, and one of them represents *interest.** 120 and 600 being *principals* are *homogeneous*, and of course form one pair—$7.20—expressing an *interest* will, with the required interest, its homogeneous term, form the other pair.

6th ex. How many feet of board, 9 inches broad, must I buy, to make the floor of a room 10 feet square?

Questions of this sort frequently perplex pupils; for they see only two terms, 9 inches and 10 feet. Their embarrassment sometimes arises from their ignorance of the nature of a square: whenever it is the case, they should be told that in a square the length is equal to the breadth, and that a floor 10 feet square, means a floor 10 feet long, and 10 feet broad. A square always represents two dimensions: length and breadth. What then is a carpet 8 yards square? A board 1 foot square? A bandbox 7 inches square? A table 18 inches square?

When able to discover 3 terms in the above question, 2 expressing feet, and the other inches, the student would be wrong to conclude that the 2 numbers of feet form a pair:

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* An explanation of the words *capital* or *principal* and *interest* may not be superfluous; for, I have met with many pupils who, altho' familiar with the *words*, had no correct ideas of their *meaning*.

A *capital* or *principal* is a sum of money lent.

The *interest* is the sum to be paid, by the person who borrows, for the use of the money lent him, on the same principle that a man pays for the hire or use of a coach, a gig, &c.—Interest is generally calculated at the rate of 5, 6, 7, &c. per cent. per annum, which means that, at the end of 1 year or 12 months you will receive 5, 6, 7, &c. dollars, for every 100 lent at the beginning. If, for 100 you receive 6, for 200 you will receive twice 6=12—for 300 three times 6=18, &c.—For further illustration, see Interest.

they are of the same name, but not *homogeneous;* for one of them expresses breadth, and the other length. The length required, with the length given, will then form a pair; while the two given breadths form the other.*

When able to point out the pairs accurately, the pupil, (keeping in view that every proportion is composed of 2 ascending, or of 2 descending ratios,) should attend to the following DIRECTIONS for

Stating the Rule of three.

1st. Let the unknown quantity be represented by x; find out its denomination, then its homogeneous term, and set them down as the first ratio.

There now remains the second ratio, or the other pair of homogeneous terms to set down: to do it with accuracy,

2d. Find out that second pair of homogeneous terms, and see whether the answer depends on the *greater*, or *less* term.

3d. Determine, from the nature of the question, whether x is to be *greater* or *less* than its homogeneous term; that is, whether the ratio first put down, is to be *descending* or *ascending*. (See the following note.)

4th. Let the other ratio be likewise descending or ascending.

NOTE.—A considerable degree of attention is requisite to determine, from the nature of the question, whether the answer is to be greater, or less, than its homogeneous term. We will give some explanation calculated to facilitate that point.

When the 1st ratio is down, there are yet 2 homogeneous terms to set down; examine whether *the term particularly related to the answer, or connected with it, or on which that answer depends,* is greater or less than the other; this being ascertained, if the term related to the answer be greater, ask yourself—will *more* bring or require *more,* or *less?* if the term related to the answer be less, ask yourself—will *less* bring or require *less,* or *more?*

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* The pupil should be taught to discriminate or distinguish accurately, the homogeneous terms forming the pairs; because this knowledge naturally leads to the *reasoning* upon which the stating of the question is founded. Should the above examples be found insufficient, select some of those hereafter inserted, or any other, at pleasure, to inculcate that knowledge.

If, in either case, the answer be *more*, it shows that the 1st term will be greater than the 2d and, of course, that the 3d must be greater than the 4th; or that the first ratio being descending, the second must likewise be descending. Put it down accordingly.

If, on the contrary, the answer be *less*, it shows that the 1st term will be less than the 2d, and that, of course, the 3d must be less than the 4th; or in other words, that the 1st ratio being an ascending one, the 2d must be ascending likewise. Put it down accordingly.

In the first example above. What is the value of 15 yards of muslin at 56 cents for 3 yards?

I place x cts. for 1st term, and 56 cts. its homogeneous term for the second. Now, I want to find whether the *term particularly related to the answer is* greater or less than the other: but which is the term particularly related to the question?—As I want to know the price of 15 yards, of course, 15, and not 3, is the term particularly related to the answer, or on which that answer depends; and as 15 is greater than 3, the other term, I say to myself: will *more yards require more, or less money?* The answer *more*, shows that the 1st term is to be greater than the 2d, or that the 1st ratio is descending; the 2d ratio must likewise be descending; therefore I put down 15 : 3; and the proportion is x : 56 : : 15 : 3.. simplified x : 56 : : 5 : 1; and as, in order to find one extreme, the product of the *means* is to be divided by the other known *extreme;* performing the operations, I find x=\$2.80.

2d ex. How many men will in 16 days do what 28 men did in 12 days?

Putting down x m : 28 m—its homogeneous term for 1st ratio, and perceiving that out of the 2 remaining numbers, 16 days and 12 days, the *greater* is particularly related to the answer, I say: *will more days require more, or less men?* The answer is evidently *less*, for the more days there are, the less men are required. The 1st term x, will then be less than the 2d, 28; and the 1st ratio being ascending, the 2d must be so likewise; of course, the proportion is x m : 28 m : : 12 dys : 16 dys.—simplified it is x : 28 : : 3 : 4 and more simplified x : 7 : : 3 : 1; hence x= 21 men.

127. When stated, to find the 1st term or answer, multiply the 2d and 3d terms together, and divide their product by the known extreme, or 4th term.

128. The product of the multiplication is always of the same denomination as the 2d term; and as this product be-

comes the dividend, the quotient and remainder are of the same denomination as that 2d term or product.

Before multiplying the means together, the 2 terms of the 2d ratio must be reduced to the same denomination.

Although the manner of stating the rule of three backwards, as we are going to explain it, is not so natural as the one just given; yet, as it is practised, we insert it for the benefit of those who, having adopted it, do not wish to relinquish it.

129. Consider the unknown quantity (represented by the letter x) as the 4th or last term; select for the 3d the homogeneous given term: this forms the 2d ratio.

There now remains the 1st ratio, or the 2 other homogeneous terms to set down: to do it with accuracy, see, from the nature of the question, whether the 4th term or x is to be *greater* or *less* than the 3d; that is, whether the 2d ratio is *ascending* or *descending*.

If ascending, as the 1st ratio must be ascending likewise, place the two remaining terms so as to have an ascending ratio. On the contrary, if descending, as the 1st ratio must be descending likewise, place the two remaining terms, so as to have a descending ratio.

The manner of determining whether the 4th term is greater than the 3d—is as follows:

Examine whether the *term on which the answer depends* is greater or less than the other; this being ascertained; if the term related to the answer be greater, ask yourself, will *more* bring or require *more*, or *less?* If the term related to the answer be less, ask yourself, will *less* bring or require *more*, or *less?*

If in either case, the answer be *more*, it shows that the 4th term will be greater than the 3d, and of course, that the 2d must be greater than the 1st—or that the 2d ratio being ascending, the 1st must likewise be ascending. Put it down accordingly.

If, on the contrary, the answer be *less*, it shows that the 4th term will be less than the 3d, and that of course, the 2d must be less than the 1st; or in other words, that the 2d ratio being a descending one, the 1st *must* be descending likewise. Put it down accordingly.

What is the value of 15 yards of muslin, at 56 cents for 3 yards?—I place x cts. for 4th term, and 56 cts. its homogeneous quantity, for 3d term. Finding, by the directions given above, that the 4th term is to be greater than the 3d;

that is, that the 2d *ratio is ascending*, I conclude that the 1st *ratio must be ascending also;* or in other words, that the 2d term must be greater than the 1st: the 1st ratio will then be 3 : 15, and the whole proportion 3 y : 15 y : : 56 cts. : x cts. simplified, it becomes 1 : 5 : : 56 : x, which, by multiplying the means, give, as before found, $2.80, for the answer.*

Recapitulation for Stating.

1st. Find out the dedomination of x and its homogeneous term.

2d. Find out the other pair of homogeneous terms, and see whether the answer depends on the *greater* or *less*.

3d. Determine whether x is to be greater or less than its homogeneous term; that is, whether the ratio first put down is to be descending, or ascending.

4th. Let the other ratio be likewise descending or ascending.

EXAMPLES.

1. If 6 primers cost 25 cents; what will 36 of them cost? Ans. $1.50.

2. I bought 9 cwt. of sugar for 108 dollars; what must my neighbour give me for 1 cwt. of it? Ans. $12.

3. I paid a man 78 dollars for 10 cords of hickory wood; wanting 1 more cord, what must I pay at the same rate? Ans. $7.80.

4. If 12 books are bound for 48 cents; what must be paid for binding 168 books? $6.72.

5. If 5 dollars are paid for 15 pounds of butter; at what rate is that for 3 pounds? $1.

6. If 12 bushels of wheat are worth 18 dollars; what is the worth of 64 bushels? $96.

7. If 12 men can build a house in 48 days; in what time could 36 men build it? 16 days.

8. Sold 12 yards of cloth for $66; what is it per yard? $ 5.50.

9. If the price of 96 pounds of sugar is $8.64; what is it per pound? 9 cts.

10. 9 pence being equal to 10 cents; how many pence are there in 9 dollars, or 900 cents? 810

11. If the sun goes 360 degrees in 24 hours; in how many hours does it go 60 degrees? 4 hours.

12. When a quire of paper sells for 25 cents; how much must be paid for 20 quires, or a *ream?* $5.

* See appendix.

13. 3 pounds Pennsylvania currency being equal to 8 dollars; how many pounds in 96 dollars?

14. In a ream, or 20 quires there are 480 sheets; how many sheets in 1 quire?

15. 3 statute miles are equal to 1 marine league; how many miles in 20 leagues, or 1 degree?

16. The equator having 360 degrees, and 1 degree being equal to 20 marine leagues; how many leagues in the equator?

130. We have said (art. 126) that the terms are always given by pairs; so that, when in a question one term has no homogeneous quantity, that solitary term must not be included in the statement.

17. Ex. If a man rides 162 miles in 4 days, when the days are 15 hours long; how many days of 12 hours will he require to perform the same journey? Ans. 5 days.

The homogeneous terms are x days and 4 days; 15 hours and 12 hours; the 162 being a solitary term, without homogeneous quantity, no notice must be taken of it in the statement.

18. How many yards of carpet 3 quarters of a yard wide, at 97 cents a yard, will cover a floor 8 yards long, and 6 yards broad? Ans. 64 yds.

We have here 4 given numbers, and the one required makes 5; so that besides the 2 *pairs*, there is 1 term over, what is to be done with it? Take no notice of it, if you cannot find a homogeneous quantity for it.

EXAMPLES.

19. If 50 men can make 1000 square yards of stone wall in 12 days; how many men must be employed to do it in 3 days? 200 men.

20. 6 men have dug 70 perches of canal in 8 days; the same quantity is to be dug in 4 days; how many men must be employed to do it? 12.

21. If, when the weight of a bushel of wheat is 65 pounds, the bushel is sold for \$1.35; what must be paid for 421 bushels? \$568.35.

22. If 20 horses eat 70 bushels of oats, in 3 weeks; how many bushels will 6 horses eat in the same time? 21 bus.

23. If I give \$5 for the use of 80 dollars, for 9 months; what must I give for \$452.80, for the same time? \$28.30.

24. What will 564*l.* sterling be worth in N. England currency, when 3*l.* sterling equal 4*l.* N. England?

£752 N. E.

25. 1170*l.* Pennsylvania currency being equal to 702*l.* sterling, what are 5*l.* Pennsylvania currency equal to in pounds sterling? £3 sterling.

26. What will 128 pounds of sugar come to at 25 cents for 3 pounds?

27. If I can get 5 pounds of almonds for 1 dollar; what must I pay for 75 pounds?

28. Last year 6 reapers, reaped a field containing 20 acres in 12 hours, I want the same field reaped in 3 hours; how many reapers must I employ?

29. 3*l.* sterling being equal to 5*l.* Penn. currency, what is the value of 360*l.* sterling in Penn. currency?

30. What is the value of 240*l.* New England currency in Penn. currency when it is known that 5*l.* Penn. and 4*l.* N. England are each equal to 3*l.* sterling? £300 Penn. cur.

31. Bought 4 pieces of cloth, 5 quarters wide, at $7 for 5 yards; each piece contained 25 yards; I give $150 in payment; must the merchant return me any change? Yes $10.

32.* A person purchased 120 gallons of rum for 78 dollars, and by adding water to it, he was enabled to sell it at 60 cents a gallon without loss; how much water did he add?

33. A man worth $7,000 pays $10 taxes; what part of a dollar is it on $100? $\frac{1}{7}$ of a dollar on every 100.

34. A merchant, who owes me $750, can pay but 85 cents on 100; how much is he to pay me at that rate? $637.50.

35. How much in length, that is 4 inches broad, will make a square foot? 36 inches, or 3 feet.

36. If the whole amount of property of a county is 74,376 dollars, and the amount of the tax 3,402; what must a man pay, whose property is worth 5,040 dollars; or what is his quota? $230.53.

37. In a county in which the property amounts to 84,200 dollars, if the tax is 2,526 dollars, how many cents is it on the dollar? 3 cents.

38. A cistern 6 feet square at the bottom, and 4 feet deep, has a pipe which will empty it in 4 hours; how many pipes of the same capacity will empty it in 12 minutes, or $\frac{1}{5}$ of an hour? Ans. 20.

39. A person whose occupation is rated at 3,600 dollars,

* Ex. 32, is the same as: at 60 cents a gallon, how many gallons must I sell for 78 dollars? The answer is 130; of course, having purchased only 120 gallons, I must add 10 gallons of water.

pays 54 cents personal tax; how many cents is that on 100 dollars? 15 mills on 100 dollars.

40. Sound going at the rate of 1142 feet in one second; how many feet am I from a person whom I see hammering on the top of a house, if $2\frac{1}{2}$ seconds pass before I hear the sound of the blow? 2855.

41. Sound going 1142 feet in 1 second; if, when standing on the bank of a river, I see, on the opposite side, the flash of a cannon, and hear the report 5 seconds after; how many feet wide is the river? Is it a mile wide? Yes, and more.

42. If 1 dollar is equal to 4s. 6d. sterling, and 6s. N. England currency; what are 3*l*. sterling equal to, in N. England currency? £4 N. E. cur.

43. A lady asked me, how many yards of nankin 9 inches broad, will it take to cover 3 quilts, 10 feet long, and 9 feet broad? 120 yds.

44. A cistern containing 48 cubic feet of water may supply an engine during 3 hours; how long can it supply 10 engines, in case of fire? 18 minutes.

45. A. bought 54 gallons molasses for 36 dollars, and being obliged to sell the gallon at 60 cents, added water, so as not to lose any thing; how much did he add? 6 gal.

46. How many gallons would he add to the same molasses, to gain 3 dollars on the whole? 11 gal.

47. A person bought a piece of cloth for [illegible]0 shillings, at 15 shillings per yard; how many yards did the piece contain?

48. If a staff 4 feet long, cast a shade 6 feet long; what is the height of a steeple, whose shade, at the same time, is 198 feet? 132 feet.

49. A cistern can supply an engine for 10 hours; how many engines of the same capacity can it supply for 30 minutes?

50. If 200 dollars in 12 months bring 12 dollars interest; what sum will bring the same in 8 months? $300.

COMPOUND RULE OF THREE.

131. The rule of three becomes *double* or *compound* when it consists of more than two ratios; that is to say, when, besides the unknown quantity and its homogeneous term, (forming a constant or common ratio) there are four or six other terms, forming 2 or 3 pairs.

132 *Rule for Stating.*

1st. As in the single rule of three, place x, representing the unknown quantity, for 1st term, and its homogeneous term for the 2d: this gives the *first* or *constant ratio*, with which all the others must separately, and individually, be compared.

2d. Select a pair of terms; no matter which; and considering it as the only one, compare it with the first or constant ratio, and state it, as in the single rule of three; that is, see whether the ratio must be ascending or descending, and state it accordingly. When this pair is down, select another, and comparing it likewise with the first and constant ratio, treat it as in the single rule of three; and so on of every other pair.

When the question is fully stated, you have only one *first* ratio, while there are several *second* ratios, which must be brought to a single one; but previous to that, reduce to the lowest denomination, and simplify, if possible. After simplification:

3d. Multiply all the *second antecedents* together; the product will be a new second antecedent; multiply likewise all the *second consequents* together for a new second consequent: thus you obtain a single ratio, which, when combined with the 1st and constant one, gives a *simple proportion*, or single rule of three, of which the 1st term is unknown, but which can easily be found, by the rule already given, viz. by multiplying the 2d and 3d terms together, and dividing the product by the 4th.

EXAMPLE.

12 men working 9 hours per day, made 108 yards of cloth in 5 days; how many yards will be made by 20 men, in 7 days and 7 hours each day?

The number of yards to be made is the unknown term. It depends not only on the number of *men*, but likewise on the *days* and *hours* employed. The 1st ratio, formed by the unknown quantity x, and its homogeneous term, 108 yards, must then be compared with every individual ratio, formed by the men, hours and days. Therefore, having set down that 1st ratio x yds. : 108 yds.—I select a pair of terms; the men for instance; 12 and 20 men; and considering every thing else equal, or in other words, viewing this 2d ratio as the only one on which the answer depends, the question assumes the shape of a single rule of three, and must be treated as

such; therefore, asking myself, *will more men do more work?** The answer *yes*, showing that the 1st ratio is descending, I set down 20 : 12 for 2d ratio; and here the statement would terminate had no other condition been annexed; but as the answer depends on the *days* likewise, I take the pair of terms they offer, viz. 5 and 7 days. Now, considering *this* ratio as the only one on which the answer depends, by comparing it with the first constant ratio, I find that it must be *descending*; for more yards will be made in 7 days, (the term particularly related to the answer,) than in 5; therefore I place it under the preceding one, in this form 7 : 5. Having yet another ratio consisting of 9 hours and 7 hours, I view *this* also, as the only one on which the answer rests; and knowing that in *less hours, less work will be done;* I conclude this ratio to be *ascending;* and place it under the 2 former, as follows, 7 : 9. As I have no more terms, the statement ends, and stands thus:

$$\begin{array}{lllllll}
 & & \overset{12}{} & & \overset{4}{} & \overset{3}{} & \\
xy. & : & \cancel{108}y. & :: & \cancel{20}m. & \cancel{12}m. & \\
 & & 4 & :: & 7 & : \cancel{5} & \text{days} \\
 & & & :: & 7 & : \cancel{9} & \text{hours} \\
\hline
196=x & : & 4 & :: & 49 & : 1. &
\end{array}$$

I simplify, by dividing 20 and 5, by 5—then 12 and 4, by 4—then 108, one means, and 9, one extreme, by 9; and lastly, 12, a means, and 3 an extreme, by 3—and after multiplying the second *antecedents* together, and the second *consequents* together, so as to give it the form of a single rule of three, I obtain the simple proportion placed under the line which gives 196 for the value of x.

132. To ascertain the important use of simplifications, let the same sum be resolved without them, that is, by multiplying all the 3d terms, or 2d antecedents together, and likewise all the 4th terms, or 2d consequents together, such as they were put down at first.

EXAMPLE 2d.

12 men, working 9 hours per day, made 108 yards of cloth in 5 days; how many men working 7 hours a day, will in 7 days, make 196 yards?

This question serves to prove the preceding one. The

* In questions of this kind, it is always taken for granted that the objects spoken of are perfectly equal to each other. Here, we suppose all the men endowed with the same vigour, activity, attention, perseverance, sobriety, &c.

first or constant ratio is x men with 12 men, its homogeneous term. As it is plain that the number of men depends on the yards, days, and hours, this constant ratio must be compared with every other individual ratio furnished by the question. Therefore, I select, at random, a pair of terms 108 yards and 196 yards, and for a moment considering this ratio *as the only one on which the answer depends*, the question assumes the shape of a single rule of three, and must be treated as one. Therefore, asking, will *more yards require more men?* The answer, *yes*, showing that the 1st antecedent is greater than its consequent; or in other words, that the ratio is *descending*, I set down 196 : 108 for 2d ratio, in order to have it descending likewise Here the statement would terminate, had no other condition been annexed; but as the answer depends on the *days* also, I take the pair of terms they offer, 5 and 7 days and compare it with first and constant ratio. Now, considering this ratio as the only one on which the answer depends, I find that having *more* days, *less* men will be required; of course, in this case the number of men sought would be less than 12; the 1st ratio is then *ascending*; so must the other be; consequently, I place it under that of yards 5 : 7 days. Having yet another ratio, 9 hours and 7 hours, I view that also as the *only one* on which the answer rests; and finding that *less* hours require *more* men, I conclude that this ratio is *descending*; and place it under the two former, in this form, 9 : 7—As I have no more terms, the full statement stands thus:

$$
\begin{array}{ccccccc}
 & & & & 4 & & \\
 & & & & \cancel{28} & & \cancel{9} \\
x\text{m} & : & \cancel{12}\text{m} & :: & \cancel{196} & : & \cancel{108}\ \text{yards} \\
 & & 1 & :: & 5 & : & \cancel{7}\ \text{days} \\
 & & & :: & \cancel{9} & : & \cancel{7}\ \text{hours.} \\
\hline
x & : & 1 & :: & 20 & : & 1.
\end{array}
$$

I simplify it by dividing 12, one means, and 108, one extreme, by 12— then cutting off the two 9s—then dividing 196 and 7, by 7—and afterwards the quotient 28, and the other extreme 7, by 7.—Now, in order to give this statement the form of a single rule of three, I multiply the second antecedents together; 4×5 give 20; and as multiplying 1 by 1 is useless, I find the above proportion, $x : 1 : : 20 : 1$, which gives 20 for the value of x.

3. If 20 workmen in 35 days earn 320 dollars; what will be the wages of 15 workmen for 70 days? $480.

4. If 100*l.* principal, gain 6*l.* interest, in 12 months; what will 400*l.* gain in 7 months? £14.

5. If 400*l.* gain 14*l.* in 7 months; what is the rate per 100*l.* for 12 months? £6.

6. If the carriage of 6 hundred weight for 150 miles, cost $24.50; what must be paid for the carriage of 9 hundred weight, 64 miles, at the same rate? $15.68.

7. If 8 men build a wall 20 feet long, 6 feet high, and 4 feet thick, in 12 days; in what time will 24 men build another 200 feet long, 8 feet high, and 6 feet thick? 80 days.

8. If 24 men build a wall 200 feet long, 8 feet high, and 6 feet thick, in 80 days; how many men must be employed to build in 12 days, a wall 20 feet long, 6 feet high, and 4 feet thick? 8 men.

9. If 120 bushels of corn can serve 14 horses 56 days; how many days will 94 bushels serve 6 horses?

days $102+\frac{1}{3}$ nearly.

10. A wall to be built to the height of 27 feet, was raised 9 feet, by 12 men, in 6 days; how many men must be employed to finish the wall in 4 days? 36 men.

11. If 40 acres of grass be mowed by 8 men in 7 days; how many men will it require to mow 480 acres, in 28 days?

24 men.

12. If a regiment of 939 men, consume 351 quarters of wheat in 168 days; how many soldiers will consume 1404 quarters in 56 days? 11268.

13. If 5 men make 300 pair of shoes in 40 days; how many men may make 900 pair in 60 days? 10 men.

14. A farmer having sown 48 bushels, found that it produced 576 bushels the first year: now supposing he sows 240 bushels of grain each year for 6 years successively; how many bushels will he have reaped altogether? 17280 bushels.

15. If 1 man can in 4 days, working 8 hours per day, make 6 yards of a certain stuff 4 feet broad; how many men must be employed to make 15 yards of the same stuff, 3 feet broad, in 2 days, working 6 hours per day? 5 men.

16. If a quantity of provisions serve 1400 men 20 weeks, at 14 ounces per day each man; how many men, will the same provisions maintain for 35 weeks, at the rate of 8 ounces per day? 1400 men.

17. Suppose a person to travel 162 miles in 8 days, when

the days are 12 hours long; how many days will he be in travelling 576 miles, when the days are 16 hours long?
$21\frac{3}{8}$ days.

18. If a family of 9 persons spend $450 in 5 months; how many months would $1120, support the same family, if 5 persons were added to it? 8 months.

19. If $100 in 12 months gain $8 interest; what sum will gain $8.60 in 5 months? $258.

20. If 4 compositors, in 16 days of 12 hours, can compose 14 sheets, of 24 pages each, 44 lines in a page, and 40 letters in a line; in how many days of 10 hours, may 9 compositors compose a volume, (using the same types,) consisting of 30 sheets, 16 pages in each sheet, 48 lines in a page, and 45 letters in a line? days $14\frac{74}{77}$ or 15 almost.

VULGAR FRACTIONS.

133. We have already seen that DECIMAL FRACTIONS are *parts* of a unit; and that these parts *invariably decrease in a ten-fold proportion:* we are now going to attend to parts or fractions of a unit, the decrease of which does not offer the regularity of decimals.

134. To form a correct idea of a FRACTION, suppose any unit whatever, divided into a certain number of *equal* parts; for ex. into 2—3—7—11—20 or 100, &c. One, or more of these parts form a *fraction*. For instance, if we divide one dollar into four *equal* parts; each of these will be the fourth part of a dollar; usually called a quarter of a dollar. If I receive 1, 2, or 3 of these quarters, it will not be the whole dollar, but merely a *part* or *fraction* of it. If, instead of four, we had divided the dollar into 5 equal parts; 1, 2, 3, &c. of these fifth parts would constitute a fraction—if in 6 equal parts; 1 or more of these *sixths* would be the fraction, &c.

If we take a pie for the unit, and divide it into 7 equal parts, he who eats 1 or 2 of these parts, feasts on a fraction of a pie.

135. But should the pie be broken into *unequal* pieces; although each of them would, in fact, be a part or fraction of the whole; yet it cannot be viewed as a regular fraction; for we could not, since all the pieces are *unequal*, say that one is the half, the fourth, or any other *aliquot* part of the pie; so

that the whole quantity must be divided, not merely into parts, but into *equal parts.*

136. Now, if an object be divided into 6 equal parts, and I get one of them, I possess one sixth of the whole ; if desirous to represent my portion in figures, shall I put down 1 alone? No : it would be 1 unit. Shall I put down 6 ? No: for that would express 6 units. How then shall I manage ? It is customary to use both figures, 1 and 6, in one of the following forms 1-6, $\frac{1}{6}$. If I had 2 of those six equal parts, I would represent them by 2-6 or $\frac{2}{6}$; 3 by 3-6 or $\frac{3}{6}$, &c.

137. The 6 or lower number indicates the value, or denomination of the equal parts, and is called the *denominator.*

138. The 1, 2, 3, &c. or upper numbers, indicate the number of those parts, and are called the *numerators.*

139. In $\frac{1}{6}$, the denominator means then that an object or unit is divided into 6 equal parts, and the numerator 1 shows that I have only 1 of those 6 parts.

$\frac{2}{7}$; here, the denominator 7, shows that the unit is divided into 7 equal parts; or shows the value of each part, and the numerator 2, indicates that the number of those parts I have is 2.

$\frac{6}{7}$; $\frac{2}{5}$; $\frac{1}{3}$; $\frac{4}{9}$; $\frac{7}{11}$; $\frac{15}{16}$; $\frac{19}{21}$; $\frac{11}{15}$; $\frac{475}{842}$.

Which are the denominators of these fractions ? What do they indicate ? Which are the numerators ? and what do they represent ?

140. As long as the numerator is less than the denominator, I have only a part of the whole ; or properly a fraction ; or a *proper fraction.* It is evident that if an apple, for instance, is divided into four equal parts, and I eat only 3 of those four parts, I have not eaten the whole apple, but merely a fraction of it; expressed by $\frac{3}{4}$.

141. If now, I eat the remaining fourth, the whole apple will be gone ; for I have eaten the $\frac{4}{4}$ (4 fourths) of it. Had it been divided into 5 equal parts, and I receive them all, I have the whole apple, or the $\frac{5}{5}$ (5 fifths). If I have the $\frac{9}{9}$ (9 ninths) I have the whole. Thus, *any fraction whose numerator is equal to its denominator expresses* 1 *unit.*

EXAMPLES.

$\frac{7}{7}$; $\frac{11}{11}$; $\frac{6}{11}$; $\frac{8}{8}$; $\frac{100}{100}$; $\frac{5}{12}$; $\frac{12}{12}$; $\frac{3}{8}$; $\frac{1}{2}$; $\frac{3}{3}$; $\frac{99}{99}$;

Which of those fractions are equal to 1 unit ?

142. If, when the numerator equals the denominator, the fraction is equal to 1 unit, it must evidently be *greater* than 1 unit when the numerator is *greater* than its denominator as: $\frac{5}{4}$; which means that I have $\frac{5}{4}$ of an object, (or the object

itself) and $\frac{1}{4}$ of another object, equal to the first. When the numerator is equal to, or greater than its denominator, the fraction is termed an *improper fraction,** as $\frac{7}{4}$; $\frac{8}{5}$; $\frac{6}{6}$; $\frac{11}{9}$; $\frac{7}{5}$.

Why are those fractions called improper?

143. If I receive the quarter of an apple, and give half of it to a friend, he has received the half of $\frac{1}{4}$, or a fraction of a fraction. These are called *compound fractions.*

144. If I buy three yards and $\frac{1}{8}$ of lace, I have bought a whole number of yards and a fraction; similar expressions are called *mixed numbers.* 3 yds. $\frac{2}{6}$—\$4$\frac{3}{4}$—7 cwt. $\frac{1}{2}$—$\frac{6}{8}$—$\frac{9}{8}$—$\frac{2}{5}$—$\frac{99}{99}$—$\frac{145}{9}$—6$\frac{3}{7}$.

Which are the proper fractions and why? which the mixed numbers and why? which are the improper fractions and why?

145. If I had a pie or pound-cake to divide equally among several boys, I must, in order to give an equal portion to each, know how many there are; for if I made 5 equal portions, and there were 6 boys, one would be without any. If there were 7, and I made 8 portions, after giving one to each, the eighth would still remain; so that they would not have received the whole object. But, when I know the number of persons, and divide the whole into as many equal parts, then I can give each his allotted part. Hence, when I want to cut an object into parts, I must first know the number of parts, so as to have exactly as many as are wanted: this, put into arithmetical language is—To reduce 1 into a fraction, I must know the denominator of the fraction, and take a numerator equal to it; or else, *make the numerator equal to the given denominator;* or *multiply* 1 *by the given denominator.*

146. Since to reduce 1 into a fraction of a given denominator, you obtain the numerator by multiplying 1 by that given denominator, it follows—that to reduce any whole number into fractions of a given denominator, that whole number must be multiplied by the denominator, and the product taken for the numerator of the given denominator.†

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* I have sometimes remarked that an improper fraction, 7-5 or 13-5—for instance, would embarrass the pupil; for, they say, how can I have 7 or 13 parts of an object, divided only into 5? this judicious observation, will be made only by a pupil of reflecting mind, who will immediately perceive the propriety of these expressions, when told that, not only *one* object, but 2, 3, 4, &c. objects have been divided into equal parts, and that 7-5 means 1 whole object, and 2-5 of another;—13-5—2 whole objects and 3-5 of another, &c.

† If you had two pies to divide each into the same number of parts, so that each person should have an equal share of each pie, it is evident that each person having 2 pieces, the number of *pieces* would be double the number of

Reduce 1 into ninths. $1\times9=9$ the numerator, and the fraction is $\frac{9}{9}$.

Reduce 2 into fourths. $2\times4=8$ the numerator, and the fraction is $\frac{8}{4}$.

Reduce	2	into ninths.	How many	sixths	in	8
"	8	" fifteenths.	"	fifths	"	5
"	1	" fourths	"	thirds	"	11
"	4	" sevenths	"	halves	"	9
"	7	" halves.				

1	equals	how many	sevenths
3	"	"	twelfths
10	"	"	tenths.

147. To reduce an *improper fraction* into a mixed number.

Divide the numerator by the denominator; the quotient will express the whole number, and the remainder will be the new numerator.

EXAMPLE.

$\frac{14}{9}$—dividing 14 by 9, the quotient 1 is the whole number, and I take the remainder 5, for the new numerator of the fraction $\frac{5}{9}$.—For, $\frac{14}{9}$ is equal to $\frac{9}{9}+\frac{5}{9}$.—Now $\frac{9}{9}$ being equal to 1 we have $1+\frac{5}{9}$.

Reduce to mixed numbers, $\frac{32}{7}$ $\frac{41}{6}$ $\frac{71}{14}$ $\frac{120}{20}$ $\frac{22}{5}$ $\frac{108}{9}$ $\frac{64}{12}$

What is the whole number and the fraction contained in $\frac{83}{9}$? in $\frac{24}{7}$? in $\frac{87}{11}$? in $\frac{144}{13}$? in $\frac{122}{10}$? in $\frac{64}{8}$? in $\frac{132}{12}$.

Since, when the numerator is equal to the denominator or contains it once, the fraction may be converted into 1 unit, it is evident that the quotient of the numerator divided by the denominator, will always give the whole number: $\frac{5}{2}$—may be decomposed into $\frac{2}{2}+\frac{2}{2}+\frac{1}{2}$ or $1+1+\frac{1}{2}$ or $2+\frac{1}{2}$—by division 2)5

$2+\frac{1}{2}$.

148. *To reduce a mixed number into an improper fraction.*

A mixed number being composed of a whole number and

---

persons; if there were 3 pies, each person receiving one piece of each, the quantity of pieces would be 3 times that of the persons; if 4 pies, the number of pieces would be equal to 4 times that of the persons, and so on. Now the denominator being the *persons*, and the whole number the *quantity* of pies; *to find the numerator, multiply the whole number by the given denominator.*

of a fraction, the denominator of which is known, the operation consists,

1st. In reducing that whole number into the fraction of the given denominator, and

2dly. In adding to this new fraction, the one given at first.

RULE.

Multiply the whole number by the denominator; to the product add the numerator; take this sum for the numerator of the improper fraction, and place it over the given denominator.

Reduce $4\frac{2}{5}$ to an improper fraction, or find how many fifths in 4 and $\frac{2}{5}$?

If I knew how many fifths there are in 4, I could, by adding that number to $\frac{2}{5}$, have the required answer. My first purpose must then be, to reduce 4 into fifths; to do it, I multiply the whole number 4, by the denominator 5, and obtain a new fraction $\frac{20}{5}$ (art. 146). Here the operation would end, had I been directed only to reduce the whole number 4 into fifths; but as my object is to know how many fifths there are in 4 and $\frac{2}{5}$, or $\frac{20}{5}$ and $\frac{2}{5}$, it is evident that I must add the numerators together, and I find $\frac{22}{5}$ for result.

NOTE.—This case and the preceding one prove each other.

Reduce to improper fractions,

$7\frac{11}{14}$	$18\frac{3}{9}$
$4\frac{4}{7}$	$4\frac{2}{5}$
$8\frac{4}{17}$	$5\frac{1}{14}$
$6\frac{5}{6}$	$5\frac{1}{3}$

In $7\frac{3}{11}$ how many elevenths,
" $19\frac{16}{21}$ " twenty-firsts
" $3\frac{3}{7}$ " sevenths
" $7\frac{10}{11}$ " elevenths
" $14\frac{3}{13}$ " thirteenths.

149. If we have $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, &c. fractions in which the denominator 7 is constantly the same, but in which the numerators are different, it is evident that $\frac{2}{7}$ is greater than $\frac{1}{7}$; that $\frac{3}{7}$ is greater than either of the former, and that $\frac{4}{7}$ is the greatest; as it is the same with respect to $\frac{2}{9}$, $\frac{5}{9}$, $\frac{6}{9}$, &c. or any other fractions having the same denominator, we naturally conclude that:

150. *A fraction increases when its numerator increases; and that it becomes less when its numerator diminishes;* or more

concisely, *that a fraction depends* DIRECTLY *on its* NUMERATOR.

Is $\frac{3}{7}$ less or greater than $\frac{4}{7}$? and why?—$\frac{2}{3}$ than $\frac{1}{3}$? why?—$\frac{7}{8}$ than $\frac{3}{8}$?—$\frac{9}{13}$ than $\frac{11}{13}$?—$\frac{12}{100}$ than $\frac{42}{100}$?—Why is $\frac{5}{6}$ greater than $\frac{1}{6}$? $\frac{4}{11}$ than $\frac{1}{11}$?

151. Hence, to make a fraction 2, 3, or 4 times *greater*, we must multiply its numerator by 2, 3 or 4, without altering its denominator: for, when we multiply the numerator by 2, 3 or 4, it will express 2, 3, or 4 times more of the same parts than before; the fractional quantity must therefore be greater.

To make $\frac{4}{13}$ three times greater, I *multiply* its numerator 4, by 3, without altering its denominator 13, and I thus obtain $\frac{12}{13}$.

Make $\frac{3}{11}$ twice greater
" $\frac{9}{37}$ 3 times greater
" $\frac{5}{48}$ 7 "
" $\frac{3}{39}$ 8 "
" $\frac{9}{41}$ 4 "

How can $\frac{3}{17}$ be made 5 times greater, and why will it be greater?

How can $\frac{2}{9}$ be made 3 times greater, and why?
" $\frac{17}{100}$ 5 "
" $\frac{9}{36}$ 4 "

152. On the contrary, if we wish to make a fraction 2, 3, 4, &c. times less, its numerator must be *divided* by 2, 3, 4, &c. (provided it can be done without a remainder,) for by lessening the numerator, we shall have less of the same parts than before; and the fractional quantity will of course be less.

$\frac{8}{11}$, can be made 2, 4, or 8 times less by dividing its numerator by 2, 4 or 8; because that numerator contains exactly 2, 4 and 8; and the respective results are $\frac{4}{11}$; $\frac{2}{11}$; $\frac{1}{11}$; but $\frac{8}{11}$ cannot be made 3, 5, 6 nor 7 times less by dividing its numerator 8; because that numerator contains neither 3, 5, 6 nor 7 exactly.—(We shall shortly find out some other means of lessening a fraction.)

Make $\frac{15}{23}$, five times less—$\frac{9}{11}$, 3 times less—$\frac{14}{9}$, 7 times less—$\frac{27}{35}$, 9 times less—$\frac{18}{25}$, 6 times less.

Can $\frac{4}{9}$ be made 3 times less by altering its numerator? How can you make $\frac{4}{9}$, 2 times less? $\frac{21}{25}$, 3 times less?—$\frac{64}{71}$, 8 times less?—$\frac{70}{78}$, 14 times less.

153. Suppose a person has $\frac{1}{2}$ of a dollar; another $\frac{1}{4}$; another $\frac{1}{5}$; another $\frac{1}{8}$, &c. it is evident that he who has the $\frac{1}{8}$ has

the smallest portion, whilst the owner of the $\frac{1}{2}$ has the largest share. In all these fractions the numerator being 1, the *diminution* of the fraction must be owing to the *increase* of the denominator, and *vice versa*, the *increase* of the fraction to the *diminution* of the denominator; and that must certainly be the case; for, what does the denominator indicate? The number of parts into which the unit is divided; of course, the less parts there are, the greater will each of them be; for, if an apple be divided only into 2 parts; another equal to the first, into 3; another into 4; each $\frac{1}{2}$ will evidently be more than each $\frac{1}{3}$, $\frac{1}{4}$ &c. Whilst, it is likewise obvious, that the more parts there are, the less will each of them be; as $\frac{1}{4}$ of an apple is not so great a portion of it as $\frac{1}{3}$ or $\frac{1}{2}$, &c.*

154. Hence we conclude, that *a fraction decreases when its denominator increases;* (and vice versa) that *it increases, when its denominator decreases;* or in other words, that *a fraction depends* INVERSELY *on its* DENOMINATOR.

Is $\frac{1}{6}$ greater or less than $\frac{1}{8}$, and why?
$\frac{3}{7}$ " " " $\frac{3}{5}$ "
$\frac{9}{11}$ " " " $\frac{9}{12}$ "
Which is greater $\frac{4}{15}$ or $\frac{4}{13}$, and why?
" " $\frac{3}{7}$ " $\frac{3}{11}$ "
" " $\frac{1}{4}$ " $\frac{1}{3}$ "

155. Hence, to make a fraction 2, 3, or 4 times *less*, without changing its *numerator*, we must *multiply its denominator* by 2, 3 or 4; for, *by increasing its denominator, we lessen the fraction which depends inversely on it.*

To make $\frac{5}{6}$, 3 times less, I multiply its denominator 6, by 3, and obtain $\frac{5}{18}$.

Make $\frac{2}{9}$ 4 times less—$\frac{4}{5}$, 3 times less—$\frac{6}{7}$, 2 times less—$\frac{2}{5}$, 9 times less—$\frac{1}{8}$, 12 times less—$\frac{21}{32}$, 7 times less—$\frac{18}{25}$, 6 times less.

156. A fraction may be made 2, 3, &c. times *greater*, by diminishing or dividing its denominator by 2, 3, &c. (when the division can be made without a remainder) for, when the denominator becomes 2, 3, &c. times less, the fraction, which depends inversely on it, must, of course, increase. But if the denominator is not divisible without a remainder, the fraction must be increased *by multiplying its numerator*, as directed above (art. 151).

* If I divide an apple into 3 thirds, and then each 1-3 into 2 equal parts, I *shall then have* 6 equal parts, or the apple will be divided into 6-6, each of *which, represented by 1-6, is* evidently less than 1-3.

To make $\frac{3}{16}$, four times greater, I divide its denominator 16, by 4, and obtain $\frac{3}{4}$.

Make $\frac{3}{8}$, twice greater, $\frac{2}{7}$, 3 times greater,
" $\frac{2}{9}$, 3 times " $\frac{1}{14}$, 7 "
" $\frac{7}{12}$, 4 times " $\frac{2}{27}$, 9 "

Make $\frac{1}{6}$, 8 times greater,
" $\frac{3}{27}$, 9 "
" $\frac{1}{45}$, 7 " and give the reason why.

From paragraphs 150 and 154 we draw the general inference: that,

157. *A fraction depends* DIRECTLY *on its numerator, and* INVERSELY *on its denominator.*

This is the *fundamental principle* on which the whole theory of fractions rests. It is similar to the one laid down respecting the quotient of a division; to which a fraction has a very great analogy; and in fact, we may say that a proper fraction is nothing but a division, not *performed* but *indicated*, the numerator being the dividend, and the denominator the divisor; for $\frac{1}{6}$ means that the unit is to be divided by 6, so as to have $\frac{1}{6}$ of it—$\frac{2}{6}$ means that 2 is to be divided by 6, &c.

158. From that fundamental principle, it follows: that *the value of a fraction does not change, when* BOTH *its terms are either multiplied, or divided by the same number.* For, by multiplying the *numerator*, for instance, the fraction, which depends *directly* on it, becomes a certain number of times *greater*; but by multiplying the *denominator*, the fraction, which depends *inversely* on it, will become the same number of times less; therefore, the *increase* on one hand being made up by the *decrease*, on the other; the value of the fraction can experience no change, and consequently it remains the same.

A similar result would likewise take place, if the two terms of a fraction were *divided* by the same number; for, the diminution of the numerator would be counteracted by that of the denominator.

$\frac{1}{4}$ multiplied by 3, gives $\frac{3}{12}$; the value of which is evidently the same. For by multiplying the numerator 1, and the denominator 4, by 3, I make them both 3 times greater; and although, on account of the numerator, the fraction increases; as it decreases, by exactly the same quantity on account of the denominator, the fraction finally experiences no change. Or, had I multiplied the numerator only, I should have made the fraction 3 times greater, but when I make the denominator 3 times greater also, the fraction becomes 3 times less

If both terms of $\frac{3}{12}$ are divided by 3, the result, $\frac{1}{4}$, is equal to the first fraction.

Multiply $\frac{2}{7}$ by 4	Multiply $\frac{7}{21}$ by 5
" $\frac{3}{15}$ " 7	" $\frac{5}{13}$ " 6
" $\frac{1}{8}$ " 9	" $\frac{7}{11}$ " 8.

Divide $\frac{3}{27}$ by 3——$\frac{5}{65}$ by 5——$\frac{22}{77}$ by 11

" $\frac{4}{28}$ " 4——$\frac{9}{15}$ " 3——$\frac{27}{63}$ " 9.

Multiply $\frac{7}{21}$ by 6, and divide the result by 7

" $\frac{801}{963}$ by 8, and divide the result by 9

" $\frac{65}{145}$ by 7, and divide the result by 5.

If I divide $\frac{9}{81}$ by 3, and the result $\frac{3}{27}$ by 3 also, I obtain $\frac{1}{9}$.

$\frac{6}{15}$ being divided by 3, gives $\frac{2}{5}$.

$\frac{1}{9}$ and $\frac{2}{5}$ are respectively equal to $\frac{9}{81}$ and $\frac{6}{15}$; yet they are presented under more simplified forms, or *each is reduced to its lowest term.*

159. So that, to reduce a fraction to its lowest terms, (or to represent it more simply without changing its value) both the numerator and denominator must be divided, without remainder, by the same number.

Reduce the following fractions to their lowest terms.

$\frac{27}{36}$——$\frac{3}{6}$——$\frac{7}{49}$——$\frac{8}{100}$——$\frac{96}{108}$——$\frac{25}{125}$——$\frac{84}{153}$——$\frac{54}{432}$——$\frac{105}{112}$——$\frac{365}{400}$——$\frac{30}{50}$——$\frac{56}{84}$.

Try the numbers that will divide both terms, as directed; (art. 122) or else find the *common measure*, to divide both terms by it. (For the manner of finding that common measure, see appendix).

160. Another method of reducing a number to a fraction of a given denominator. As the quotient of a number divided by *unity* is the same as the number itself, an integer may be represented in a fractional form by writing it as a numerator, with unity for denominator; thus, $3=\frac{3}{1}$; $8=\frac{8}{1}$; in this state it may be reduced to any denominator whatever, by multiplying both its terms by the proposed denominator. Ex. Reduce 3 into sevenths.

$3=\frac{3}{1}$ which being multiplied by 7 give $\frac{21}{7}$ required answer.

MULTIPLICATION OF FRACTIONS.

161. When we multiply any number whatever by 1, the product is equal to the number multiplied, or to the multiplicand; if we multiply the same multiplicand by a number *greater* than the unit or 1, the product depending directly on *its factors,* will be *greater* than before; that is, greater than

the multiplicand. If we have the multiplicand $\frac{3}{7}$ to multiply by the unit, the product is $\frac{3}{7}$, or the multiplicand itself. If, instead of 1, we wish to multiply by 2; as this multiplier is the double of the preceding one, the product will be double likewise. Our intention is then, to make the fraction $\frac{3}{7}$, twice as great. But we already know that to make a fraction 2, 3, &c. times greater we must multiply its numerator by 2, 3, &c. Hence,

162. *To multiply a fraction by a whole number, we must multiply its numerator by that number, without altering its denominator.**

Multiply	$\frac{7}{11}$ by 2 ——	$\frac{3}{17}$ by 4 ——	$\frac{7}{45}$ by 6
"	$\frac{3}{13}$ " 4 ——	$\frac{2}{19}$ " 9 ——	$\frac{4}{26}$ " 5
"	$\frac{1}{19}$ " 8 ——	$\frac{7}{36}$ " 9 ——	$\frac{1}{4}$ " 8

163. Since the product equals the multiplicand when the multiplier is 1, it is evident, that the product must be *less* than the multiplicand if the multiplier become *less than* 1, or is a fraction; for, in such a case, the product will not be the multiplicand taken even once, but merely *a part* of it. Hence,

164. *To multiply a whole number by a fraction, multiply the whole number by the numerator and divide the product by the denominator.*

EXAMPLES.

Multiply	4	by	$\frac{3}{17}$
"	3	"	$\frac{3}{11}$
"	4	"	$\frac{17}{71}$
"	9	"	$\frac{1}{41}$
"	17	"	$\frac{3}{85}$
"	2	"	$\frac{1}{2}$
"	4	"	$\frac{1}{4}$
"	6	"	$\frac{1}{3}$

I multiply 4, by 3, the numerator, and find 12 for product; had I been directed to multiply 4 by 3, the operation would now be performed and 12 would be the true result; but 4 was not to be multiplied by 3 *only*, but by $\frac{3}{17}$, a quantity 17 times less than 3. I have then employed a multiplier 17 times too great; the product is of course 17 times greater than it ought to be; to make it 17 times less, I must divide it

* We likewise know that a fraction may be made 2, 3, &c. times greater by dividing its denominator by 2, 3, &c. when that can be done exactly; but, as this does not always happen, it has been thought adviseable to give only the rule which admits of no exception, and to notice the other in a note. 3-8 may be made twice greater by dividing its denominator 8, by 2; because the division can be done exactly, and the result is 3-4, but not so of 3-7—5-9—1-11, &c.

Multiply	$\frac{1}{6}$ by 3	$\frac{3}{10}$ by 5
"	$\frac{5}{48}$ " 6	$\frac{2}{21}$ " 7
"	$\frac{7}{36}$ " 4	$\frac{2}{45}$ " 9

by 17, or consider 17 as the denominator of 12, and the true product is then $\frac{12}{17}$.

165. When we multiply a fraction by a whole number, or the same integer by the same fraction, although the product is the same, yet the principle is different. For, in the first case, the multiplicand, which is a fraction, becomes greater, whilst in the second case, the multiplicand, which is a whole number, becomes less.

1st. $\frac{2}{3}\times3=\frac{6}{3}$ or 2 wholes. 2d. $3\times\frac{2}{3}=\frac{6}{3}=2$ wholes.

In 1st ex. $\frac{2}{3}$ becomes 2, a larger quantity; whilst 3 is reduced to 2, a less quantity, in 2d ex.

166. If I had $\frac{2}{5}$ to multiply by 2, the product would be $\frac{4}{5}$; but if instead of 2, the multiplier was $\frac{2}{3}$, a fraction 3 times less than 2, it is evident that the first multiplier 2, being 3 times too great, the first product, $\frac{4}{5}$, must also be 3 times too great. In order to reduce it to its true value, $\frac{4}{5}$ is to be made 3 times less; and that is done by multiplying the denominator 5, by 3, (art. 155) and the result is $\frac{4}{15}$. Hence,

167. To multiply one fraction by another fraction,

Multiply numerator by numerator, and denominator by denominator.

168. The *product* will evidently be less than the *multiplicand*; since the multiplier, being a proper fraction, is less than 1 unit: it is in fact the same thing as taking a part only of the multiplicand. Therefore: compound fractions, or fractions of fractions, are reduced to single fractions by the same RULE, viz. by multiplying their numerators together for a new numerator, and their denominators for a new denominator.

Multiply	$\frac{2}{3}$	by	$\frac{7}{8}$,		Take	$\frac{1}{3}$	of	$\frac{2}{3}$
"	$\frac{5}{9}$	"	$\frac{2}{5}$		"	$\frac{7}{8}$	"	$\frac{1}{7}$
"	$\frac{3}{4}$	"	$\frac{4}{11}$		"	$\frac{1}{4}$	"	$\frac{2}{5}$
"	$\frac{9}{17}$	"	$\frac{1}{3}$		"	$\frac{2}{3}$	"	$\frac{1}{5}$

Reduce to single fractions,

$\frac{1}{3}$ of $\frac{2}{3}$ and $\frac{3}{7}$ of $\frac{5}{9}$

$\frac{2}{7}$ " $\frac{9}{13}$ " $\frac{8}{9}$ " $\frac{5}{6}$

$\frac{4}{11}$ " $\frac{3}{16}$ " $\frac{7}{9}$ " $\frac{6}{7}$

$\frac{4}{9}$ " $\frac{9}{11}$.

169. When the same figures are in the numerator of one fraction, and in the denominator of the other, they may be *cancelled*, that is, cut off. The same thing may be done when *one of the terms* is an aliquot part of another: for, in both

cases, it is dividing both terms by the same number; an operation which we know can be done without altering the value of the fracton. For instance: when I have to multiply $\frac{5}{9}$ by $\frac{2}{5}$, the new numerator would be 5×2; the new denominator 9×5; or $\frac{5}{9}\times\frac{2}{5}$. We can, without altering the value of the fraction, divide both terms by 5, and the result is $\frac{2}{9}$: a result which could have been obtained by at once cutting off 5 from each of the above fractions $\frac{5}{9}$ and $\frac{2}{5}$, and 2 would have remained for numerator, and 9 for denominator.

Multiply $\frac{4}{7}$ by $\frac{1}{4}$ Take the $\frac{2}{7}$ of $\frac{2}{3}\frac{1}{2}$
" $\frac{3}{8}$ " $\frac{8}{11}$ " $\frac{4}{9}$ " $\frac{9}{16}$

Reduce to single fractions, $\left\{\begin{array}{l} \frac{4}{7} \text{ of } \frac{2}{4}\frac{1}{4} — \frac{7}{9} \text{ of } \frac{6}{7} \\ \frac{5}{8} \text{ " } \frac{1}{2}\frac{6}{5} — \frac{1}{1}\frac{1}{4} \text{ of } \frac{3}{22} \end{array}\right.$

170. To multiply a *mixed* number by a fraction, or by another mixed number, reduce the mixed numbers into improper fractions, (art. 148) and multiply them as proper fractions.

Multiply $5\frac{2}{7}$ by $\frac{3}{8}$—$5\frac{2}{7}$ reduced to an improper fraction is $\frac{37}{7}\times\frac{3}{8}=\frac{111}{56}$.

Multiply $\left\{\begin{array}{l} 3\frac{1}{4} \text{ by } \frac{2}{5} — 4\frac{5}{7} \text{ by } \frac{12}{7} — 2\frac{3}{5} \text{ by } 3\frac{1}{4} — 7\frac{3}{8} \text{ by } 1\frac{5}{16} \\ 4\frac{2}{9} \text{ " } \frac{16}{18} — 7\frac{2}{9} \text{ " } \frac{8}{3} — 14\frac{1}{2} \text{ " } 5\frac{2}{3} — 9\frac{4}{7} \text{ " } 3\frac{1}{2}. \end{array}\right.$

171. Compound fractions must be reduced to single ones, previous to multiplying them.

Multiply $\frac{2}{3}$ of $\frac{1}{7}$ by $\frac{1}{5}$—$\frac{6}{7}$ of $\frac{2}{5}$ by $\frac{6}{7}$ of $\frac{3}{7}$—$\frac{3}{4}$ of $\frac{4}{5}$ by $\frac{2}{7}$ of $\frac{7}{8}$
" $\frac{7}{9}$ " $\frac{3}{8}$ " $\frac{3}{8}$—$\frac{3}{7}$ " $\frac{2}{3}$ " $\frac{4}{5}$ " $\frac{5}{8}$—$\frac{1}{5}$ " $\frac{10}{11}$ " $\frac{3}{8}$ " $\frac{4}{5}$.

DIVISION OF FRACTIONS.

172. When we divide any number whatever by 1, the quotient is equal to the dividend. When the divisor increases, the quotient diminishes, and of course it will be *less* than the dividend; on the contrary, when the divisor is less than 1, or is a fraction, the quotient will be *greater* than the dividend.

When we have to *divide a fraction by a whole number*, (as, from what has just been said, the quotient will be *less* than the dividend) our intention must be to make the fraction less, and that is done, as already taught, either by dividing its numerator, by the whole number, when that can be performed without remainder; or, by multiplying its denominator. As this last operation is always possible, we will say, that:

173. *To divide a fraction by a whole number, the denominator must be multiplied by that whole number.* To divide $\frac{3}{7}$ by 4, I multiply its denominator by 4 and obtain $\frac{3}{28}$.

Divide $\frac{1}{3}$ by 2—$\frac{1}{2}$ by 3—$\frac{6}{11}$ by 9
" $\frac{5}{8}$ " 6—$\frac{5}{6}$ " 8—$\frac{3}{8}$ " 2
" $\frac{9}{11}$ " 4—$\frac{4}{9}$ " 6—$\frac{2}{3}$ " 4
" $\frac{7}{8}$ " 5—$\frac{1}{6}$ " 7—$\frac{12}{17}$ " 3.

174. If I had to divide $\frac{3}{4}$ by 7, the quotient would be $\frac{3}{28}$; but if, instead of 7, the divisor was $\frac{7}{8}$, or a quantity 8 times less, the 1st quotient $\frac{3}{28}$, would be 8 times too small, since it was obtained by a divisor 8 times too great; I must, of course, make it 8 times greater, so as to have the true result; and to do it, I merely multiply the numerator 3, by 8, and have for real quotient $\frac{24}{28}$, or $\frac{6}{7}$. The operation then, is performed by multiplying the *numerator* of the 1st fraction, by the *denominator* of the 2d, and *its* denominator by the numerator. Hence we draw this rule:

175. *To divide a fraction by another, invert the terms of the divisor and perform the multiplication of fractions* (as directed art. 167.)

$\frac{1}{4}$ divided by $\frac{2}{3}$—the inverted divisor is $\frac{3}{2}$ to be multiplied by $\frac{1}{4}$, the result is $\frac{3}{8}$.

Divide $\frac{3}{9}$ by $\frac{2}{8}$——$\frac{4}{7}$ by $\frac{2}{5}$——$\frac{2}{3}$ by $\frac{7}{8}$
" $\frac{5}{11}$ " $\frac{15}{17}$——$\frac{4}{5}$ " $\frac{5}{6}$——$\frac{7}{8}$ " $\frac{3}{5}$
" $\frac{2}{9}$ " $\frac{5}{7}$——$\frac{3}{4}$ " $\frac{8}{9}$——$\frac{1}{4}$ " $\frac{1}{2}$
$\frac{7}{8}$ " $\frac{2}{3}$——$\frac{1}{3}$ " $\frac{1}{4}$.

Cancel, or simplify, as directed in multiplication (art. 169).*

176. Hence it appears that when we divide a fraction by another, we intend to make it at once as many times *greater* as is indicated by the denominator of the divisor, and as many times *less* as is indicated by its numerator. So that whenever the divisor is a proper fraction, as the denominator is greater than its numerator, the divided fraction increases more than it diminishes, and the quotient is, of course, greater than the dividend.

177. If I had 2 to divide by $\frac{5}{6}$; the divisor being *less* than 1 unit, the quotient must be *greater* than the dividend 2. My intention is then, to make the whole number 2, as many times *greater* as is indicated by the denominator 6, and as many times *less*, as is indicated by the numerator 5: hence, *I multiply the whole number by the denominator and divide the product by the numerator, which now becomes denominator.* Thus, 2 *multiplied by 6=12;* this product 12, being divided by the

* When the Numerators are the same, the quotient will have for its Numerator the denominator of the divisor, and for its denominator the denominator of the dividend.

$\frac{281}{4682179} \div \frac{281}{348392} = \frac{348392}{4682179}$.

numerator 5, which is thus converted into a denominator, gives $\frac{12}{5}$ for quotient, or $2+\frac{2}{5}$.

178. Should we give to the whole number 2, the unit for denominator, (an operation which does not change its value, since any number whatever, divided by 1, gives a quotient equal to the number itself;) we should then have $\frac{2}{1}$ to divide by $\frac{5}{6}$; that is, a fraction to divide by another. To do it, we must invert (art. 175) the terms of the divisor $\frac{5}{6}$, which gives $\frac{6}{5}$, and multiply the other fraction $\frac{2}{1}$ by it; the product is as before $\frac{12}{5}$. Hence we draw this new

RULE.

179. *To divide a whole number by a fraction, place 1 for the denominator of the whole number, invert the terms of the divisor and multiply.*

Divide 2 by $\frac{1}{3}=6$	6 by $\frac{7}{8}$	1 by $\frac{1}{4}$	7 by $\frac{1}{8}$	4 by $\frac{1}{3}$
" 4 " $\frac{2}{3}$	5 " $\frac{1}{9}$	2 " $\frac{1}{6}$	10 " $\frac{1}{10}$	6 " $\frac{3}{10}$
" 9 " $\frac{3}{5}$	$\frac{3}{9}$ " $\frac{2}{5}$	3 " $\frac{2}{7}$	1 " $\frac{1}{100}$	2 " $\frac{4}{5}$

180. To divide *mixed numbers*, reduce them into improper fractions before you begin the division.

1. Divide $4\frac{2}{7}$ by $\frac{2}{9}$
2. " $3\frac{4}{7}$ " $1\frac{2}{5}$
3. " $16\frac{9}{31}$ " $8\frac{2}{7}$
4. " $6\frac{3}{8}$ " $8\frac{3}{5}$

I reduce 4 into $\frac{28}{7}$ and adding the $\frac{2}{7}$ given, find $\frac{30}{7}$ for the improper fraction, to be divided by $\frac{2}{9}$. I invert the divisor $\frac{2}{9}$, which becomes $\frac{9}{2}$, and multiplying, I find $\frac{270}{14}$, or simplified, $\frac{135}{7}$. Dividing the numerator by the denominator, to obtain the whole number, I find $19+\frac{2}{7}$ for final result.

4th Ex. After reducing the dividend $6\frac{3}{8}$, and the divisor $8\frac{3}{5}$ to improper fractions $\frac{51}{8}$, and $\frac{43}{5}$, I invert the divisor $\frac{43}{5}$, which becomes $\frac{5}{43}$, and multiplying, I obtain $\frac{255}{344}$ for quotient, or nearly $\frac{3}{4}$.

181. Compound fractions must be reduced to single ones, previous to performing the division.

1. Divide $\frac{2}{3}$ of $\frac{5}{7}$ by $\frac{1}{9}$ of $\frac{3}{11}$
2. " $\frac{1}{6}$ " $\frac{2}{13}$ " $\frac{3}{5}$ " $\frac{1}{21}$
3. " $\frac{3}{8}$ " $\frac{11}{41}$ " $\frac{2}{17}$ " $\frac{3}{19}$
4. " $\frac{1}{6}$ " $\frac{3}{5}$ " $\frac{4}{5}$ " $\frac{7}{8}$.

I first reduce the dividend into the simple fraction $\frac{10}{21}$, then the divisor into $\frac{3}{99}$, and dividing I find $\frac{990}{63} = \frac{110}{7} = 15+\frac{5}{7}$.

ADDITION OF FRACTIONS.

192. If I had $\frac{1}{4}$ and $\frac{1}{5}$ to add together, I could not say that they were equal to $\frac{2}{4}$, nor $\frac{2}{5}$, for neither would be correct, since fourths and fifths not being equal, cannot be added together;

but if I have $\frac{2}{7}$ and $\frac{3}{7}$ to add together, I at once see that they are equal to $\frac{5}{7}$, because those given quantities express the same portion of one unit. Hence it seems, that fractions can be added together, only when their denominators are the same; or when they have a common denominator. Then, before we perform an addition of fractions, it is necessary to reduce them to a common denominator, without, however, altering their value. Let us see how that can be done.

183. Add $\left.\begin{array}{r}\frac{3}{8}\\ \text{and } \frac{4}{7}\end{array}\right\}$ Not being able to add those fractions under this form, I must endeavour to change them into others of the same value, but with a *common denominator*. But, what number shall I select for that common denominator? Will it be 10, 20, 40, &c. or any other number taken at random? No, for I could bring neither $\frac{3}{8}$ nor $\frac{4}{7}$ to express tenths, nor twentieths; and although I could reduce $\frac{3}{8}$ into fortieths, by multiplying it by 5, yet I could not do the same of $\frac{4}{7}$. What number must I choose then? 56; that is, the product of both denominators. For as 56 contains the denominator 8, 7 times, I can bring the $\frac{3}{8}$ into 56ths by multiplying the numerator 3, and the denominator 8, by 7; an operation which does not alter the value of the fraction $\frac{3}{8}$, (as proved art. 158) it, of course, becomes $\frac{21}{56}$. Since the denominator 7, is contained 8 times in 56, if I multiply $\frac{4}{7}$ by 8, I shall obtain $\frac{32}{56}$, without changing its value. I then have two new fractions, $\frac{21}{56}$, and $\frac{32}{56}$, equal to the given ones, but having both a common denominator 56, and whose sum is $\frac{53}{56}$.

$$\left.\begin{array}{r}\frac{3}{8}\times7=21\\ \frac{4}{7}\times8=32\end{array}\right\}56$$
$$\overline{53}$$

184. When there are more than two fractions, both terms of each must be multiplied by the product of all the denominators except its own; so as to have for common denominator, the product of *all* the denominators. But as it is useless to multiply those denominators together more than once, the rule we give is:

185. *Multiply each numerator by the product of all the denominators except its own, to obtain new numerators; and the common denominator will be the product of all the given denominators.*

The operation is performed as follows: place under each other all the given fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{7}$, $\frac{2}{9}$, as per margin.

$\frac{1}{2}$ $\frac{2}{3}$ $\frac{4}{7}$ $\frac{2}{9}$ Since each numerator is to be multiplied by all the denominators except its own, the first, 1, must be multiplied by 3, by 7, and by 9: the second, 2, by 2, by 7, and by 9; the third, by 2, by 3, and by 9; the fourth, by 2, by 3, and by 7; the operation then assumes the following form.

$$\left.\begin{array}{l} \frac{1}{2}\times3\times7\times9=189 \\ \frac{2}{3}\times2\times7\times9=252 \\ \frac{4}{7}\times2\times3\times9=216 \\ \frac{2}{9}\times2\times3\times7=\ \ 84 \end{array}\right\}378$$

$$9\times2\times3\times7 \mid 741$$

Now, multiplying the first numerator 1, by 3=3; that product by 7 =21; and this by 9=189. This 189 is the 1st new numerator, which I place opposite the other. The 2d numerator is obtained by multiplying 2 by 2=4; this by 7=28; and 28 by 9=252. In the same manner are found the 3d=216 and the 4th=84. Now, multiplying all the denominators, 2, 3, 7 and 9 together, I find 378 for common denominator, and place it on the right of the *brace* which includes the numerators.

All the fractions being now of the same kind, I can, by summing up their numerators, know how many parts there are altogether. This addition shows that I have $\frac{741}{378}$, or simplified $\frac{247}{126}=1+\frac{121}{126}$. Hence, to add fractions together,

RULE.

186. 1st. Reduce them to the same denominator, (usually called common denominator). 2d add up their new numerators together, and 3d, if you obtain an improper fraction, reduce it to a mixed number.

EXAMPLES.

Add $\frac{1}{4}$ and $\frac{3}{5}$	Ans.$=\frac{17}{20}$	Add $\frac{1}{8}$, $\frac{3}{10}$ and $\frac{4}{12}$
$\frac{1}{2}$, $\frac{3}{4}$ and $\frac{5}{8}$	$=1+\frac{7}{8}$	$\frac{7}{9}+\frac{3}{7}+\frac{5}{8}+\frac{2}{3}$
$\frac{7}{8}$, $\frac{4}{6}$, $\frac{6}{10}$ and $\frac{6}{7}$	$=3+\frac{23}{280}$	$\frac{2}{3}+\frac{4}{5}+\frac{7}{8}+\frac{5}{6}$
$\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$ and $\frac{7}{8}$	$=2+\frac{7}{8}$	$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}$
$\frac{4}{5}$, $\frac{1}{2}$, $\frac{5}{6}$ and $\frac{2}{8}$	$=2+\frac{23}{60}$	$\frac{2}{11}+\frac{3}{13}+\frac{5}{16}$.

$\frac{1}{2}+\frac{1}{3}+\frac{1}{6}$.

NOTE.—*Simplify*, if possible, before reducing to the same denominator.

187. If, among the given quantities, to be added together, there are some whole numbers, do not reduce them to fractions, but keep them, to add to the result the fractions give. Add:

$$\frac{7}{9},\ 8,\ \frac{3}{7},\ \frac{5}{11},\ -2,\ \frac{8}{11},\ 6,\ \frac{9}{17},\ -\frac{14}{15},\ \frac{7}{8},\ 5,\ \frac{2}{3},\ \frac{1}{2},\ 3.$$

188. If *mixed numbers* are given, keep the whole number

such as it is, to be added to the result, but reduce the accompanying *fraction* to the common denominator.

EXAMPLES.

$2+\frac{2}{3}, \frac{5}{6}, \frac{9}{11}, 1+\frac{3}{7}$
$\frac{4}{9}, \frac{3}{4}, 2+\frac{17}{18}, 2+\frac{4}{5}$
$6+\frac{1}{4}, 2, \frac{5}{8}, 3+\frac{7}{16}$

In the first example, I reduce the fractions $\frac{2}{3}, \frac{5}{6}, \frac{9}{11}$ and $\frac{3}{7}$ to the same denominator, without altering the whole numbers $2+1=3$, which must be added to the sum obtained by the fractional part.

189. *Improper fractions* may be treated as *proper fractions* and reduced to a common denominator; or, which is preferable, they may be reduced to mixed numbers, and added as in the preceding rule.

EXAMPLE.

$\frac{1}{6}, \frac{3}{2}, \frac{24}{15}, \frac{4}{5}$

$$\left.\begin{array}{r} \frac{1}{6}\times 5=5 \\ 1+\frac{1}{2}\times 15=15 \\ 1+\frac{3}{5}\times 6=18 \\ \frac{4}{5}\times 6=24 \end{array}\right\} 30$$

$$\frac{62}{30} \quad =2+\frac{1}{15}$$

Add the unreduced $1+1=2$

Total amount $4\frac{1}{15}$.

190. In this example, seeing that 30 is a multiple of all the denominators, I select it for common denominator, and instead of multiplying each numerator by the product of all the denominators except its own, I multiply each numerator by the number of times its denominator is contained in 30. For instance: I multiply the first numerator 1, by 5, because its denominator 6, is contained 5 times in 30.—The second numerator by 15, because its denominator 2 goes 15 times into 30.—The third and fourth by 6, because 30 contains their denominator 5, 6 times.

The selection of a number containing all the denominators, shortens considerably the operation, and should be resorted to, whenever practicable.

EXAMPLES.

Add $\frac{8}{15}, 2+\frac{5}{12}, \frac{29}{20}, \frac{13}{6}$ 60 for common denominator.
$\frac{92}{56}, \frac{1}{12}, 4, 3+\frac{6}{9}, \frac{47}{15}$ 1260 do.
$\frac{3}{8}, \frac{5}{6}, 4+\frac{2}{3}, —\frac{4}{5}, \frac{3}{2}, \frac{19}{15}, 2+\frac{1}{12}$
$\frac{9}{7}, \frac{31}{28}, 3\frac{5}{11}, \frac{11}{8}, —\frac{6}{10}, \frac{42}{27}, \frac{3}{18}, \frac{2}{3}$.

There is a cistern which has 4 pipes; the first will empty it in 10 minutes; the second in 20; the third in 40; and the fourth in 80; if they should all be set running at once, what

part of the cistern will be emptied in 1 minute; and in how many minutes will the whole cistern be emptied?

$\frac{3}{16}$ of the cist. in 1 m. The whole, in $5.33\frac{1}{3}$ minutes.

SUBTRACTION OF FRACTIONS.

191. As objects of *different kinds* can neither be added nor subtracted, it is evident, that in order to subtract fractions, they must be reduced to the *same denominator*. When this is done,

RULE.

192. Subtract the less numerator from the greater, and under the remainder place the common denominator.

Take $\frac{1}{2}$ from $\frac{3}{4}$. Under this form I could not subtract, or at least, give a name to the remainder, but after they are both reduced to $\frac{2}{4}$ and $\frac{3}{4}$, I see that 1, the difference of the numerators must be $\frac{1}{4}$.

Take	$\frac{2}{5}$	from	$\frac{5}{7}$	Ans. $\frac{11}{35}$
"	$\frac{3}{7}$	"	$\frac{1}{2}$	$\frac{1}{14}$
"	$\frac{4}{9}$	"	$\frac{2}{3}$	$\frac{2}{9}$
"	$\frac{5}{12}$	"	$\frac{3}{5}$.	

If from	$\frac{1}{2}$ a yard I take	$\frac{1}{3}$,	what will remain?	Ans. $\frac{1}{6}$.
"	$\frac{5}{8}$	"	$\frac{1}{4}$	do.?	
"	$\frac{9}{16}$ of a dollar		$\frac{2}{5}$	do.?	
"	$\frac{3}{4}$	"	$\frac{2}{3}$	do.?	

193. When several fractions are to be subtracted from several others, sum up each set, separately, so as to have only one fraction to take from another; reduce these to a common denominator, and subtract the numerators, as directed above.

Take $\frac{2}{3}$ and $\frac{4}{7}$ from $\frac{3}{4}$ and $\frac{5}{6}$.—I reduce the first set to $\frac{26}{21}$, the second to $\frac{19}{12}$, and reducing these two last to a common denominator, I find $1+\frac{60}{252}$, to take from $1+\frac{147}{252}$, which leaves $\frac{87}{252}$ or $\frac{29}{84}$.

Take $\frac{3}{10}$ and $\frac{2}{7}$, from $\frac{8}{9}+\frac{3}{4}$

$\frac{1}{7}+\frac{3}{11}$ " $\frac{3}{8}+\frac{4}{7}$

$\frac{2}{9}+\frac{5}{12}$ " $\frac{5}{8}+\frac{6}{9}$

From $\frac{9}{12}+\frac{5}{6}$ take $\frac{6}{15}+\frac{2}{7}+\frac{4}{11}$

$\frac{6}{7}+\frac{3}{4}+\frac{7}{9}$ " $\frac{1}{4}+\frac{2}{5}-\frac{3}{8}$ Ans. $2\frac{277}{2520}$.

$\frac{1}{2}+\frac{5}{6}$ " $\frac{4}{5}-\frac{5}{9}$.

194. When a *mixed number* is to be taken from another, reduce the accompanying fractions to the same denominator, and if the fraction in the subtrahend is greater than the other,

as in that case it cannot be subtracted, it becomes necessary to borrow 1 from the whole number; that 1 is to be converted into a fraction, the denominator of which, must be the common denominator; add this new fraction to the smaller one; subtract the greater from their sum, and count the integer as 1 less.

Take $1.\frac{5}{6}$ from $2\frac{1}{3}$

$2.\frac{2}{6}$
$1.\frac{5}{6}$
———
$0.\frac{3}{6}$ or $\frac{1}{2}$.

I first reduce $\frac{1}{3}$ to $\frac{2}{6}$, in order to have a common denominator, so that I have, as in the margin, from 2 and $\frac{2}{6}$ to take $1.\frac{5}{6}$; but as $\frac{5}{6}$ cannot be taken from $\frac{2}{6}$, I borrow 1 from the integer 2. That 1, is to be converted into a fraction; but what denominator shall I give that fraction? naturally the common denominator 6, and I say $1=\frac{6}{6}$; to these $\frac{6}{6}$ adding the smaller fraction $\frac{2}{6}$, I find $\frac{8}{6}$. From that sum $\frac{8}{6}$, taking $\frac{5}{6}$, the remainder is $\frac{3}{6}$ or $\frac{1}{2}$ for the fractional part, and as I borrowed 1 from the 2, I must count it only as 1, and subtracting the lower one from it, no integer remains; so that the difference is $\frac{1}{2}$. Or, use this rule: After reducing the accompanying fractions to the same denominator, if the subtrahend be greater, take its numerator from the common denominator, and to that difference add the numerator of the other fraction, for the true numerator of the remainder, and count the integer as 1 less.

EXAMPLE.

From $2\frac{1}{3}$ take $1\frac{5}{6}$

From $2\,\frac{2}{6}$.
Take $1.\frac{5}{6}$
———
$0.\frac{3}{6}$ or $\frac{1}{2}$.

The two accompanying fractions being reduced to a common denominator, I have, as per margin, $1.\frac{5}{6}$ to take from $2.\frac{2}{6}$.—But as the subtrahend $\frac{5}{6}$ cannot be subtracted from $\frac{2}{6}$—I take 5, its numerator, from the common denominator 6, and to the difference 1, adding 2, the numerator of the other fraction, I obtain 3 for the real numerator of the remainder $\frac{3}{6}$ or $\frac{1}{2}$, and counting the integer 2 as 1 less, I find 0 integer.

Take	$1\frac{7}{8}$	from	$3\frac{3}{4}$	Ans. $1.\frac{7}{8}$	Take	$1.\frac{5}{6}$	from	$1.\frac{14}{18}+\frac{1}{6}$
"	$1\frac{2}{7}$	"	$1\frac{1}{3}$	$0.\frac{1}{21}$	"	$2\frac{1}{4}+\frac{1}{9}$	"	$4.\frac{3}{8}$
"	$2\frac{1}{3}$	"	$3\frac{1}{4}$	$0.\frac{11}{12}$	"	$\frac{7}{5}+\frac{1}{15}$	"	$5.\frac{7}{12}$
"	$\frac{11}{8}$	"	$1\frac{9}{24}$	0.				
"	$2\frac{26}{45}$	"	$5\frac{32}{41}$					
"	$6\frac{15}{19}$	"	$13\frac{8}{9}$.					

195. A *fraction*, may be subtracted from an *integer*, by sub-

tracting the *numerator* of the fraction, from its *denominator*, for the new numerator; the denominator remaining the same; and diminishing the integer by 1.

EXAMPLES.

Take $\frac{5}{8}$ from 7.

$8-5=3$. numerator. $\frac{3}{8}$ fraction.

Integer $7-1=6$.

Final result $6\frac{3}{8}$.

From 8 the denominator, take the numerator 5, the remainder 3 becomes the new numerator, under which I place the same denominator 8, so that the fractional part is $\frac{3}{8}$ and the integral part 6—final result $6\frac{3}{8}$.

From 4 take $\frac{3}{7}$.	Ans. $3\frac{4}{7}$	From $7\frac{1}{4}$ yds. take $2\frac{1}{2}$ yds.	
" 6 " $\frac{5}{8}$.		" 6 " $\frac{7}{8}$	
" 2 " $\frac{1}{2}+\frac{2}{5}$.		" 3 " $\frac{14}{17}+\frac{3}{8}$.	

Having already said (art. 157) that a fraction is nothing but an indicated division; or that it may be considered as the remainder of a division, the numerator of which is the remaining dividual, and the denominator the divisor; it follows, that like the remainder of a division, we may

196. *Reduce a Vulgar Fraction into Decimals,*

By annexing to the *numerator* as many zeros as there are decimal classes wanted, and by dividing it by the denominator, and counting off, in the quotient, as many figures as there were zeros added to the numerator.

197. When the division of the numerator by the denominator is without a remainder, the decimal fraction will be exactly equal to the vulgar fraction; but when something remains, find more decimal classes.

EXAMPLE.

Reduce $\frac{3}{8}$ into decimals: 4 classes.

8)3 0000
———
0.3750.

To the numerator 3, I annex 4 zeros, the number of classes required. I then divide 30000 by the denominator 8; the quotient is 3750; and now as I must cut off 4 figures, that is, as many as I annexed noughts, the decimal fraction will be 0.3750.

In fact, by adding 1, 2, 3, &c. zeros to the numerator, I make it 10, 100, &c. times too great, but when instead of units, I make the quotient express *tenths, hundredths,* &c. I counterbalance the former increase, and consequently do not alter the value of the fraction.

Reduce $\frac{1}{16}$ to decimals: 4 classes.

$$\begin{array}{r} 16)10000 \\ \hline 625 \end{array} \quad \text{Ans. } 0.0625.$$

The quotient is only 625, but, as I must have 4 decimals, I place 0 before the 6, and a dot on its left thus: 0.0625.

EXAMPLES.

With 4 decimals—$\frac{4}{7}$, ans. 0.5714+—$\frac{4}{85}$ ans. .0470+—$\frac{5}{8}$—$\frac{1}{3}$ of $\frac{3}{5}$—$\frac{2}{5}$ of $\frac{3}{8}$—$\frac{1}{75}$ of $\frac{3}{4}$.

With 3 decimals—$\frac{1}{2}$—$\frac{1}{20}$—$\frac{1}{25}$—$1+\frac{4}{5}$—$2+\frac{3}{8}$—$\frac{7}{8}$ of $\frac{3}{4}$—$\frac{12}{7}$—$\frac{774}{8746}$.

NOTE.—Simplify whenever it is possible.

198. *To Reduce a Decimal Fraction to a Vulgar Fraction of a given denominator.*

Multiply the decimals by the given denominator, and take the integers of the product for the numerator of the given denominator.

EXAMPLE.

1. 0.75 to be reduced into 8ths.

$$\begin{array}{r} 0.75 \\ 8 \\ \hline 6.00 \end{array} = \frac{6}{8}.$$

I multiply 75 by 8; from the product 600, I separate 2 decimals, and take the integer 6, for numerator of 8, the given denominator; so that the vulgar fraction is $\frac{6}{8}$.

199. As by neglecting all the decimals of the product, the numerator is sometimes diminished; in order to counteract that diminution, increase the *integers* by a unit, whenever the figure expressing the tenths is 5 or above it.

EXAMPLE.

2. Reduce 0.75 into 9ths.

$$\begin{array}{r} 0.75 \\ 9 \\ \hline 6.75 \\ 6+1=\frac{7}{9} \end{array}$$

The product of 0.75 by 9 is 675; and the integer, after separating 2 decimals, would be 6, but as the figure 7, representing the tenths, is above 5, instead of taking only 6 for numerator, I take 6 increased by 1, or 7, and have $\frac{7}{9}$ for the vulgar fraction.

It is true that $\frac{7}{9}$ is a little more than 0.75; but the difference between them is less than it would have been had I taken only $\frac{6}{9}$, as may easily be seen by reducing both $\frac{7}{9}$ and $\frac{6}{9}$ to decimals. The 1st=0.777, the 2d=0.666—0.77 exceeding 0.75 only by 2 hundredths; while 0.66 is less than 0.75 by 9 hundredths. And the reason is obvious, for 0.75 is nearer 1 than 0; of course, by neglecting it entirely the di-

minution is $\frac{3}{4}$; while by considering 0.75 as 1, the increase is only $\frac{1}{4}$.

3d. Reduce 0.75 into 10ths. The product of 75 by 10 is 750. The decimals off, $\frac{7}{10}$, if the 50 hundredths are entirely neglected; or $\frac{8}{10}$, if the integer is increased by 1 on account of neglecting 5 tenths. The increase is usually adopted.

$$\begin{array}{r} 10 \\ \hline 7.50, \end{array}$$

$\frac{7}{10}$ or $7+1=\frac{8}{10}$

4th. Reduce 0.75 into 11ths. The product of 75 by 11 is 825, which gives $\frac{8}{11}$, by entirely leaving off the 25 hundredths, the 2 representing the tenths being less than 5.

$$\begin{array}{r} .75 \\ 11 \\ \hline 8.25 \end{array}$$ or $\frac{8}{11}$

Reduce	0.87 into	27ths.	Reduce	0.047 into	83ds.
"	0.48 "	5ths.	"	1.56 "	74ths.
"	2.70 "	16ths.	"	0.66 "	15ths.

200. The above reduction is founded on the following reason. When I multiply the decimals by the given denominator, I make them a certain number of times greater, but when afterwards, I consider the integers, not as units, but as parts of units, indicated by the same denominator, I then counterbalance the former increase, and of course, no alteration ensues.

201. If I had 0.06 to reduce to 4ths, for instance, the product of 6 by 4 would be 24, and as I must separate 2 decimals, there will be no integer, consequently no numerator. Hence I conclude, that the decimal fraction 0.06, cannot be reduced into 4ths; and in fact, it is evident that 6 hundredths cannot contain even $\frac{1}{4}$ of a unit.

$$\begin{array}{r} 0.06 \\ 4 \\ \hline 0.24=\frac{0}{4} \end{array}$$

202. We have already seen that a decimal number is a fraction decreasing in a ten-fold ratio; we may here add, (having treated of vulgar fractions) that any decimal number whatever, may be put under the form of a vulgar fraction, by taking the decimals for numerator; and for denominator, the unit, followed by as many ciphers as the given number contains decimal classes. For instance:

0.1 is the same as $\frac{1}{10}$—0.01 the same as $\frac{1}{100}$—$2.14=2\frac{14}{100}$.
0.2 " $\frac{2}{10}$—0.06 " $\frac{6}{100}$—$0.096=0\frac{96}{1000}$.
0.7 " $\frac{7}{10}$—1.004 " $1\frac{4}{1000}$.

In every one of which, the numerator is expressed by the significant decimal figures, and the denominator 1, followed by as many zeros as there are decimal classes in the given number.

Put under the form of vulgar fractions, the following examples.

0.70	0.8000	1.0405	12.005
1.0485	5.0008	0.4026	7.074
0.01075	0.08008	2.5505	0.20054.

When under the form of vulgar fractions, they may be simplified.

SIMPLE REDUCTION.

203. If I change 1 dollar for 100 cents, or for 4 quarters, or for 16 five-penny bits, (Penn. cur.) I have neither gained nor lost by the exchange, but have merely received objects of a different name or denomination, for the one I gave away. This is usually called receiving the change of 1 dollar in cents, in quarters, &c.; but in *arithmetical language*, it is called, *reducing* a dollar into cents, quarters, &c.

204. Reduction consists then, in changing objects of one denomination into objects of another denomination, without altering their value.

As reduction depends much on the knowledge of the following tables, we here present them.

FEDERAL MONEY.

1 dollar	equals	10 dimes or 100 cents or 1,000 mills,
1 dime	"	10 cents or 100 mills,
1 cent	"	10 mills.

205. Since 1 dollar is equal to 10 dimes, 100 cents, or 1000 mills, it is evident that the number of dimes must be 10 times that of the dollars; the number of cents, 100 times that of the dollars; and the number of mills, 1,000 times that of the dollars; consequently, to reduce

Dollars into dime;	multiply	the dollars	by	10,	or annex	1 zero.
Dollars into cents	"	"	"	100,	"	2 zeros.
Dollars into mills	"	"	"	1,000,	"	3 zeros.

206. We have already seen in treating of fractions, (art. 146) that to reduce a whole number into halves, thirds, fourths, &c. it must be *multiplied* by 2, 3, 4, &c. of course, to reduce

dollars, dimes, cents, &c. into halves, thirds, fourths, &c. they must respectively be *multiplied* by 2, 3, 4, &c.

Reduce \$4, to dimes. Ans. 40.
\$7, to dimes.
\$14, to cents. 1400.
\$17, to mills. 17000.
4 dimes to mills. 400.
315 cents to mills.

What part of 1 dime is 1 cent?
In 3 dollars how many cents?
In 45 dollars how many dimes?
In 8 dimes how many cents?
In 17 cents how many mills?
What part of 1 dollar is 1 cent?
In 427 dollars how many cents?
What part of 1 cent is 1 mill?

207. We have seen that by removing the dot or point (art. 84) in a decimal number 1, 2, 3 places to the right, we make the number 10, 100, 1000 times greater; consequently, when a number of dollars and cents is given, to reduce them to cents, the only thing to be done is, to remove the point. Ex. \$7.25 cents, is 725 cents without the dot. When thus brought to cents, reduce to mills or to halves, thirds, &c. as already directed.

Reduce \$17.06 to cents.	\$172.05 to mills.
141.17 to cents.	25.99 to $\frac{1}{3}$ of mills.
2.49 to mills.	6.9 to mills.
19.56 to $\frac{1}{2}$ cents.	0.24 to $\frac{1}{4}$ of mills.
4. to $\frac{1}{4}$ cents.	2.06 to $\frac{1}{2}$ of cents.

208. If 1 dollar is equal to 100 cents, how many cents are there in 17 dollars?

This is nothing but a single rule of three: x : 100 cts. : : \$17 : \$1—in which x=1700 cents.

209. All operations on reduction partake of the nature of the rule of three. They can of course, offer no difficulty; but as it is customary to give some rules, we will follow the usual method.

210. By *integer* is meant the principal unit or highest denomination. In the table of federal money, 1 dollar is the integer, principal unit, or highest denomination. But if we have a number of dimes and cents, and no dollars, as: 3 dimes and 4 cents, the *dimes* may be looked upon as the integer.

211. One denomination is *higher* than another, when it is

of greater value; and *lower*, when it is of less value. Thus, a shilling is higher than a penny, and lower than a pound.

212. SIMPLE REDUCTION is the changing of the *integer* into its own lower denominations; or the changing of the lower denominations to the integers.

213. The changing of a *higher* to a *lower* denomination is called *descending reduction*; this is done by *multiplication*.

214. The changing of a *lower* to a *higher* denomination, is called *ascending reduction*, this is done by *division*.

215. *Rules for Descending Reduction.*

1st Rule. Multiply the integer by the number it contains of the next lower denomination.

ENGLISH MONEY.

1 pound, *l.* is equal to 20 shillings,
1 shilling or *s.* " 12 pence,
1 penny or *d.* " 4 quarters (*qrs.*) or farthings.

Reduce 6*l.* to shillings. $xs. : 20s. :: 6l. : 1l.$—$6\times20=120s.$

216. NOTE 1.—To reduce pounds to shillings, multiply them by 20, because 1 pound is equal to 20 shillings.*

1.	Reduce	14	pounds	to shillings.	Ans. 280.
2.	"	216	"	"	4320.
3.	"	19	"	"	380.
4.	"	847	"	"	16940.

5. What part of 1 pound is 1 shilling?

217. NOTE 2.—To reduce shillings to pence, multiply by 12, because 12 pence are equal to 1 shilling.

1. Reduce 4 shillings to pence. $xd. : 12d. :: 4s. : 1s.$
$x=12\times4=48d.$

2.	"	17	"	"	Ans. 204.
3.	"	19	"	"	228.
4.	"	45	"	"	540.
5.	"	96	"	"	1152.
6.	"	240	"	"	2880.

7. What part of 1 shilling is 1 penny?

218. NOTE 3.—To reduce pence to farthings or quarters,

---

* This reduction may be considered as changing a whole number into a fraction of a given denominator; for, to reduce 3 pounds for instance, into shillings, is the same as reducing them into twentieths; since 1 shilling is the 20th of 1 pound, and we find that 3 pounds are equal to 60-20ths, or 60 shillings.

multiply by 4, because 1 penny is equal to 4 farthings or quarters.

1. Reduce 3 pence to quarters. $xq.:4q.::3d.:1d.\ 4\times3=12$.

2. Reduce 1 penny to quarters.		Ans. 4.
3. " 2 pence to	"	8.
4. " 9 "	"	36.
5. " 38 "	"	152.
6. " 47 pence to farthings.		188.
7. " 145 "	"	580.
8. What part of 1 penny is 1 farthing?		$\frac{1}{4}$.

219. 2d Rule. If there are several intermediate denominations between the integers and the lowest one required, multiply successively by the number each denomination contains of the one immediately lower, till you reach the required denomination; or in other words, reduce the highest to the next lowest, this to the next, this last to the next, and so on.

1. Reduce 7 pounds to pence.

I reduce first, 7 pounds to shillings, by multiplying by 20, and find 140 shillings—and 2dly, those 140 shillings into pence, by 12=1680 pence.

$$\begin{array}{rl} 7 & \text{pounds} \\ 20 & \\ \hline 140 & \text{shillings} \\ 12 & \\ \hline 1680 & \text{pence.} \end{array}$$

2. In 81 pounds how many shillings, and how many pence? Ans. 19440d.

3. 1 pound	"	"	"	240d.
4. 243 pounds	"	"	"	58320d.
5. Reduce 37 pounds to pence.				8880.
6. " 37 pounds to farthings.				35520.
7. " 1 pound to farthings.				960.
8. " 74 shillings to farthings.				3552.
9. " 3 pence to farthings.				12.
10. " 3 shillings to pence.				36.
11. In 1 shilling how many pence?				12.
12. In 1 shilling how many farthings?				48.
13. One penny is what part of 1 pound?				$\frac{1}{240}$.
14. One farthing is what part of 1 shilling?				$\frac{1}{48}$.
15. One farthing is what part of 1 pound?				$\frac{1}{960}$.
16. In 3 shillings how many farthings?				144.

220. 3d Rule. If several denominations are given, to be reduced to the lowest of these, or to a lower one; after reducing the highest to the next one, add to the product those of this denomination already given; reduce this sum to the next denomination; add the given ones, and so on.

1. Reduce 7*l.* 6*s.* to shillings.

```
 7l. 6s.
 20
 ---
140
  6
 ---
146
```

I reduce the 7 pounds to shillings, and obtain 140 shillings, and as 6 are already given, I add them to the product 140 and finally obtain the sum 146.

The multiplication and addition are usually made at once, as:

```
 7l.  6s.
  20
 ---
 146 shillings.
```

2. Reduce 9*l.* 14*s.* to pence.

```
 9l. 14s.
 20
 ---
 194
  12
 ---
2328
```

I first reduce the 9*l.* 14*s.* to shillings as directed above, and then the shillings to pence, by multiplying by 12.

3. In 11*l.* 4*s.* how many shillings and pence? 224*s.*—2688*d.*
4. In 11*l.* 4*s.* how many farthings? 10752 farthings.
5. In 4*s.* how many farthings? 192 qrs.
6. In 4*s.* 8*d.* how many farthings? 224.
7. In 4*s.* 8$\frac{3}{4}$*d.* how many farthings? 227.
8. In 1*l.* 4*s.* how many farthings? 7872.
9. In 244*l.* 13*s.* 6*d.* how many farthings? 234902.
10. Reduce 17*l.* 16*s.* 9*d.* to pence. 4281.
11. " 174 " 18*s.* 10*d.* to farthings. 167944 qrs.
12. " 99 " 19*s.* 11$\frac{3}{4}$*d.* to farthings. 95992 qrs.
13. " 25 " 5*s.* 3$\frac{1}{4}$*d.* to pence. 6063$\frac{1}{4}$.
14. " 0 " 6*s.* 9*d.* to pence. 81d.
15. " 0 " 6*s.* 9*d.* to farthings. 324 qrs.
16. " 0 " 17*s.* 11$\frac{1}{2}$*d.* to quarters. 862.
17. 1 penny is what part of 7*l.* 4*s.* $\frac{1}{1728}$.
18. 1 farthing is what part of 4*s.* 3*d.* $\frac{1}{204}$.
19. 1 shilling is what part of 15*l.* 15*s.* $\frac{1}{315}$.

20. Reduce 42 French crowns at 8s. 3d. Pennsylvania currency to pence. 4158.

21. Reduce 42 French crowns at 6s. 8d. Virginia currency to pence. 3360.

22. Reduce 42 French crowns at 8s. 9d. New York currency to pence. 4410.

23. In 15 guineas at 28s. New England currency, how many six-pences? 840.

24. In 15 guineas at 21s. 9d. South Carolina currency, how many pence? 3915.

25. In 15 guineas at 35s. Pennsylvania currency, how many six-pences and farthings? 1050 six-pences—25200qrs.

WEIGHTS.

221. AVOIRDUPOIS WEIGHT.

This serves for objects of a coarse nature, and all metals except silver and gold.

1 Ton (T.)	equals	20 hundred weight *cwt.*
1 cwt.	- -	4 quarters, or 112 *lb.*
1 qr.	- -	28 pounds *lb.*
1 lb.	- -	16 ounces *oz.*
1 oz.	- -	16 drams *dr.*

16 drams	make	1 ounce *oz.*
16 oz.	-	1 *lb.*
28 lb.	-	1 *qr.*
4 qrs.	-	1 *cwt.*
20 cwt.	-	1 *ton.*

222. N. B.—Although the name *hundred weight* leads to suppose that this weight contains exactly 100 pounds, yet the student will perceive that it contains 112 lb. since it is equal to 4 quarters, each of 28 lb. Coffee and a number of other articles are bought and sold by the nett hundred, which contains only 100 lb.—so that a ton of coffee weighs only 2000 lb.; whilst a ton of sugar weighs $112\times20=2240$ lb.

1. Reduce 24 tons to cwt. $24\times20=480$.
2. Reduce 6 quarters to pounds. $6\times28=168$lb.
3. In 7 pounds how many ounces? 112 oz.
4. Reduce 16 ounces to drams. 256 dr.
5. What part of 1 ton is 1 cwt.?
6. Reduce 142 cwt. 2 qrs. to ounces. 255360 oz.
7. In 9 quarters how many drams? 64512 dr.
8. Reduce 47 lb. 12 oz. to drams. 12224.

9.	Reduce 47 qrs. to pounds.	1316lb.
10.	Reduce 6 T. 13 cwt. to quarters.	532 qrs.
11.	Bring 3 qrs. 20 lb. 14 oz. to drams.	26848 dr.
12.	What part of 1 cwt. is 1 ounce?	$\frac{1}{1792}$.
13.	Reduce 4 T. to ounces.	143360 oz.
14.	How many drams in 1 quarter?	7168 dr.
15.	What part of 1 quarter is 1 dram?	$\frac{1}{7168}$.
16.	How many pounds in 1 ton?	2240 lb.
17.	In 25 qrs. 6 lb. how many ounces?	11296.
18.	What part of 1 pound is 1 dram?	$\frac{1}{256}$.
19.	How many pounds in 1 cwt?	112 lb.
20.	142 lbs. equal how many ounces?	2272 oz.
21.	1 ounce is what part of 1 ton?	$\frac{1}{35840}$.
22.	What part of 4 tons is 1 quarter?	$\frac{1}{320}$.
23.	In 14 cwt. 0 qr. 25 lb. how many pounds and ounces?	1593 lb. 25488 oz.
24.	74 lb. 2 oz. 15 dr. make how many drams?	18991 dr.

REDUCTION DESCENDING.

223. TROY WEIGHT.

1 Pound (*lb.*)	equals	12 ounces *oz.*
1 oz.	- -	20 pennyweights *dwt.*
1 dwt.	- -	24 grains *gr.*

By this, jewels, gold, silver* and liquors are weighed.

1 lb. avoirdupois is equal to 14 oz. 11 dwt. and nearly 16 grains troy.

1.	Reduce 8 pounds to ounces.	$8\times12=96$ oz.
2.	Reduce 17 pounds to ounces.	204 oz.
3.	In 9 ounces how many pennyweights?	180.
4.	1 pound contains how many grains?	5760.
5.	1 grain is what part of 1 pound?	$\frac{1}{5760}$.
6.	145 dwts. equal how many grains?	3480.
7.	In 1 pound how many pennyweights?	240.
8.	In 1 ounce how many grains?	480.
9.	1 grain is what part of 1 ounce?	$\frac{1}{480}$.
10.	Reduce 7 pennyweights to grains.	168.

* Gold is divided into *carats*, by which is understood the 24th part of any quantity.

22 carats of gold and 2 carats of alloy is the standard for gold.

The standard for silver coin is 11 ounces and 2 pennyweights of silver, and 18 pennyweights of alloy.

11. What part of 6 pennyweights is 1 grain ? $\frac{1}{144}$.

12. How many grains in 1 spoon weighing 2 oz. 7 dwts. 10 grs. ? 1138.

13. In 1 ingot of gold weighing 1 lb. 7 oz. how many grains ? 9120 grs.

224. APOTHECARY WEIGHT.

By this weight, apothecaries mix their medicine, but buy and sell by avoirdupois weight.

1 pound (℔)	equals	12 ounces ℥*
1 ℥	- -	8 drams ʒ
1 ʒ	- -	3 scruples ℈
1 ℈	- -	20 grains (*gr.*)

225. Note.—The pound troy and pound apothecary weight are the same; the subdivisions only are different.

1. In 4 pounds, how many ounces ? 48.
2. Reduce 78 ℔ to drams. 7488.
3. 1 pound equals how many scruples ? 288.
4. 1 scruple is what part of 1 pound ? $\frac{1}{288}$.
5. In 7 ℥ how many drams and scruples ? 56 ʒ and 168 ℈.
6. Reduce 11 ounces to ℈ and grains ? 5280 gr.
7. 1 ℔ equals how many grains ? 5760.
8. 1 ℥ equals how many grains ? 480.
9. Reduce 7 drams to scruples. 21.
10. 1 grain is what part of 1 dram ? $\frac{1}{60}$.
11. 1 dram equals how many grains ? 60.
12. Reduce 7 ℔ 4 ℥ to scruples. 2112.
13. In 6 scruples how many grains ? 120.
14. 1 scruple is what part of 1 ounce ? $\frac{1}{24}$.
15. In 147 ℔ how many drams ? 14112.

226. LONG MEASURE.

1 circle equals 360 degrees *deg.*
1 degree (*deg.*) equals 69½ statute miles

1 M.	-	8 furlongs *fur.*
1 fur.	-	40 poles *P.*
1 P. rod or perch	-	5½ yards *yds.*
1 yd.	-	3 feet *ft.*
1 foot	-	12 inches *in.*
1 in.	-	3 barley corns.*

* 1 league is equal to 3 miles. 1 degree is equal to 60 geographic miles.

227. To multiply by ½, take half of the multiplicand and add it to the product—so 6×5½ is

	6		7
	5½		69½
	30		483
½ of 6	3	½ of 7	3½
	33		486½

1. Reduce 3 degrees to geographic miles. 180.
2. In 18 degrees, how many statute miles? 1251.
3. Bring 7 furlongs to perches. 280.
4. Bring 1 degree to furlongs. 556.
5. Bring 14 yards to feet, 42.
6. 1 degree is what part of a circle? $\frac{1}{360}$.
7. Bring 1 mile to feet. 5280.
8. In 4 furlongs, how many inches? 31680.
9. Reduce 4 perches to feet. 66
10. 1 foot is what part of 1 pole? $\frac{2}{33}$.
11. Reduce 1 circle to geographic miles. 21,600.
12. Reduce 4 miles to yards. 7040.
13. 1 foot is what part of 1 mile? $\frac{1}{5280}$.
14. 1 yard is what part of 1 mile? $\frac{1}{1760}$.
15. Reduce 1 circle to statute miles and then to yards and feet. 132,105,600 ft.
16. What part of 1 circle is 1 yard? 44,035,200th.
17. Bring 7 yards to barley corns. 756.
18. Bring 1 circle to barley corns. 4,755,801,600.
19. In 18 yards, how many barley corns? 1,944.
20. Reduce 2 miles 5 furlongs to yards. 4,620.
21. How many inches round the globe which is 360°? 1,585,267,200.

1 hand is equal to 4 inches. This is used for the height of horses.

1 fathom is equal to 6 feet, and is used to ascertain the depth of water.

How many feet high is a horse 13 hands high? A horse 15 hands high?

A horse 11 hands high? another 12 hands high?

A river 8 fathoms deep, is how many feet deep?—A harbour of 3 fathoms?—a well 4 fathoms?

228. CLOTH MEASURE.

1 yard equals	4 quarters.	$2\frac{1}{4}$ inches	make	1 nail.
1 qr. -	4 nails.	4 nails	-	1 qr. of a yd.
1 na. -	$2\frac{1}{4}$ inches.	4 quarters		1 yd.

1 ell Flemish equals 3 quarters of an American yard.
1 ell English - 5 - - - - -
1 ell French - 6 - - - - -

1. Reduce 10 yards to quarters. 40.
2. Bring 1 quarter to inches. 9.
3. 1 inch is what part of 1 quarter? $\frac{1}{9}$.
4. Reduce 3 nails to inches. $6\frac{3}{4}$.
5. 1 ell Flemish equals how many nails? 12.
6. Reduce 7 English ells to quarters. 35.
7. Bring 3 ells French to inches. 162.
8. Reduce 1 yard to inches, 36.
9. 1 inch is what part of 1 yard? $\frac{1}{36}$.
10. How many nails in 17 ells English? 340.
11. 1 nail is what part of 1 ell English? $\frac{1}{20}$.
12. 1 nail is what part of 1 ell French? $\frac{1}{24}$.
13. Bring 3 yards 2 quarters to nails. 56.
14. Reduce 27 ells Flemish 1 quarter to inches. 1467.

229. LAND MEASURE.

1 mile square	equals	640 acres square *a.*
1 acre	-	4 roods *r.*
1 rood	-	40 perches *p.*
1 perch	-	$30\frac{1}{4}$ yards *yd.*
1 yard	-	9 feet *ft.*
1 foot	-	144 inches *in.**

This measures all things in which length and breadth only, are considered. To multiply by $\frac{1}{4}$, take $\frac{1}{4}$ of the multiplicand and add it to the product.

1. Reduce 4 miles to acres. 2560.
2. Bring 7 acres to square perches. 1120.
3. How many yards in 12 acres? 58080.
4. Bring 14 yards square, to inches. 18144.
5. Reduce 1 square mile to yards and feet square. 27,878,400 ft.

* Many students have an erroneous idea of this measure: they are generally acquainted with the figure, but not with the manner of ascertaining its contents, or area. If asked, for instance, the contents of a room 10 feet square, they frequently answer 40 feet: because, they say, each side being 10 feet

6. 1 square foot is what part of 1 square mile?
7. Bring 3 roods to yards square. 3630.
8. In 36 perches, how many square feet? 9801.
9. 1 foot is what part of 1 perch?
10. 1 yard is what part of 1 acre?
11. In 3 acres and 2 roods, how many yards square? 16940.
12. Bring 8 yards to square inches. 10368.
13. Bring 4 perches and 6 yards to square inches. 164592.
14. 1 square foot is what part of 1 square yard?
15. Reduce 3 roods, 28 perches, and 7 yards to square feet. 40356.
16. 1 square inch is what part of 1 square yard? $\frac{1}{1296}$.
17. How many inches in 1 square perch? 39204.
18. Bring 34 perches, 8 yards, 6 feet to square inches. 1,344,168.

230. The best way of measuring land is, by a surveyor's chain of 4 poles, divided into 100 equal parts, called *links*. 1 pole being 16½ feet, the 4 poles=66 feet or 22 yards.

1 chain being equal to 4 poles; 1 square chain will be equal to 16 square poles, and as 1 square acre contains 160 square poles, to find how many square chains 1 acre contains, we must divide 160 square poles by 16 square poles, and the

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long, the 4 sides must be 4 times 10=40 feet. To give them a correct idea of this measure, mark out on a slate or a black board, a foot square: divide the 4 sides each into 12 inches and by eleven horizontal lines divide it, into 12 rectangles (or long squares) being each 1 inch wide and 12 inches long, (see fig. 1) if now we divide one of these rectangles (as in fig. 2) by drawing 11 vertical or perpendicular lines, we shall have 12 small squares, each being 1 inch wide and 1 inch long; or, 1 inch square; and since there are 12 of these rectangles, each being divisible into 12 inches square, the whole figure will evidently contain 12 times 12, or 144 inches square. So that to find the area of a square or a rectangle, multiply the length by the breadth or height.

FIG. 1.

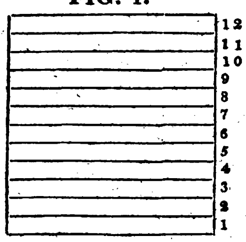
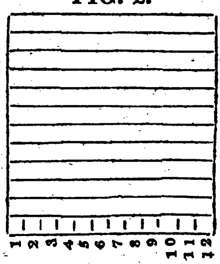


FIG. 2.



quotient 10 is the number of square chains in 1 acre squa
that is, 10 chains long and 1 wide, make 1 acre, or 222 ya
by 22, or 666 feet by 66.

MEASURE OF LENGTH.	SQUARE MEASURE
1 chain=4 poles or 100 links.	1 acre=10 sq. chain
1 pole=25 links.	1 sq. c=16 sq. poles
1 link= 7. 92 in.	

1. Reduce 4 chains to poles and links.

4	4
4	100
16 poles.	400 links.

2. Reduce 3 poles to links. 75 lir
3. Reduce 18 links to inches. 142. 56
4. In 9 chains how many inches? 7128
5. 1 inch is what part of 1 chain? 7
6. Reduce 4 acres to square chains. 40 sq. cha
7. Reduce 74 square chains to square poles. 1184 sq. po
8. Reduce 10 acres to square poles. 1600 sq. po
9. 1 square pole is what part of 1 acre. 1
10. Reduce 7 acres 7 chains to poles square. 1232 sq

231. CUBIC OR SOLID MEASURE.

This serves to measure all things in which length, brea
and thickness are considered.

1 cord of wood is { 128 solid feet (i. e.) 8 long, 4 broad and 4 high.

1 ton or load { 40 feet of round timber, 50 feet of hewn timber.

1 solid yard 27 solid or cubic feet.
1 solid foot 1728 solid inches.

1. Reduce 4 cords to solid feet. 5
2. 21 solid yards to solid feet. 5
3. 17 feet to solid inches. 293
4. 1 solid foot is what part of 1 cord? 1
5. 1 solid inch is what part of 1 yard? 1/466
6. Reduce 3 yards 8 feet to inches. 1537
7. Bring 4 tons round timber to inches. 2764
8. Bring 5 cords 2 yards to feet. 6

1728
27
12096
3456
46656.

232. LIQUID MEASURE.

1 tun (4 hogsheads)	equal	2 pipes
1 pipe	-	2 hhds.
1 hhd.	-	63 gallons.
1 gallon	-	4 quarts.
1 qrt.	-	2 pints.
1 pint	-	4 gills.

4 gills	make	1 pint.
2 pts.	-	1 quart.
4 qrts.	-	1 gallon.
63 gall.	-	1 hogshead.
2 hhds.	-	1 pipe or butt.
2 pipes, or butts.		1 tun *T.*

In Pennsylvania 16 gallons make ½ a barrel—31½ gallons 1 barrel—64 gallons 1 double barrel—84 gallons 1 puncheon—42 gallons 1 tierce.

In New England, 10 gallons make an anchor of brandy (anc.)—18 gallons 1 runlet (run.).

NOTE.—1 pint, wine measure, is 28⅞ cubic inches—1 quart is 57¾—1 gallon 231.

1.	Reduce 4 tuns to pipes.	$4\times2=8$.
2.	Reduce 12 hhds. to gallons.	756.
3.	Reduce 6 gall. to pints.	48.
4.	Reduce 14 barr. to quarts.	1764.
5.	Reduce 1 tun to pints.	2016.
6.	In 3 tierces how many quarts?	504.
7.	In 4 hhds. how many gallons?	252.
8.	In 9 tuns how many pints?	18144.
9.	1 pint is what part of 1 tun?	$\frac{1}{2016}$.
10.	Bring 3 half barrels to pints.	384.
11.	Bring 9 gallons to gills.	288.
12.	1 gill is what part of 1 gallon?	$\frac{1}{32}$.
13.	1 pint is what part of 1 puncheon?	$\frac{1}{672}$.
14.	Bring 7 double barrels to pints.	3584.
15.	Bring 3 pipes, 2 tierces, and 1 anchor to quarts.	1888.
16.	Bring 5 punch. 2 bar. and 1 runlet to pints.	4008.
17.	1 runlet and 1 half barrel to gills.	1088.
18.	6 pipes, 2 tierces, 2 barrels to gallons.	903.
19.	6 anc. 5 gal. 2 qrts. to pints.	524.

233. DRY MEASURE.

This measure is used for grain, fruit, salt, &c.

1 bushel (*bu.*) is 4 pecks.
1 peck (*pe.*) is 8 quarts.
1 quart 2 pints *pts.*

2 pints	make	1 quart (*qrt.*)
8 quarts	-	1 peck *pe.*
4 pecks	-	1 bushel *bu.*

A bushel is $18\frac{1}{2}$ inches wide and 8 deep, containing $2150\frac{2}{5}$ cubic inches.

1.	Reduce 18 bushels to pecks.	72.
2.	Bring 14 pecks to quarts.	112.
3.	In 7 quarts how many pints ?	14.
4.	Bring 7 bushels to $\frac{1}{2}$ pecks.	$56\frac{1}{2}$.
5.	Reduce 6 pecks to pints.	96.
6.	1 pint is what part of 1 bushel ?	$\frac{1}{64}$.
7.	1 quart is what part of 1 bushel ?	$\frac{1}{32}$.
8.	Reduce 11 bushels 3 pecks to pecks.	47.
9.	Bring 7 pecks 6 quarts to pints.	124.
10.	In 8 bushels 3 quarts, how many pints ?	518.

NEW ENGLAND DENOMINATIONS.

1 last (*last*) makes	2 weys.	1 bushel makes	4 pecks.
1 wey (*wey*) -	5 quarters.	1 peck (*pk.*)	2 gallons.
1 chaldron (*ch.*)	4 qrs.	1 gallon	2 pottles.
1 qr.	2 cooms.	1 pottle (*pot.*)	2 quarts.
1 coom (*co.*)	2 strikes.	1 qt.	2 pints.
1 strike (*str.*)	2 bushels.		

A gallon contains $268\frac{4}{5}$ cubic inches.

1.	Reduce 4 chaldrons to quarters.	16 qrs.
2.	Bring 3 lasts to quarters.	30 qrs.
3.	" 3 quarters to bushels.	24 bus.
4.	" 6 bushels to gallons.	48 gal.
5.	" 3 pecks to pottles.	12 pot.
6.	" 14 gallons to pints.	112 pts.
7.	" 1 last to gallons.	640 gal.
8.	" 6 chaldrons to bushels.	192.
9.	In 4 quarters how many pecks ?	128 pks.
10.	1 bushel is what part of 1 chaldron ?	$\frac{1}{32}$.
11.	1 gallon is what part of 1 last ?	$\frac{1}{640}$.
12.	Bring 3 lasts, 2 qrs. 1 bus. to gallons.	2056 gals.
13.	" 7 bus. 3 pks. 1 gal. to pottles.	126 pot.

14. Bring 7 pecks 6 quarts to pints. 124 pts.
15. " 8 pecks 1 quart to pints. 130 pts.

234. TIME.

1 year is 12 months, or* 52 weeks 1 day, 6 hours;
or 365 days 6 hours.

1 week	7 days.
1 day	24 hours.
1 hour	60 minutes.
1 min.	60 seconds.

60 seconds (*sec.* or ″)	make	1 minute *min.*
60 minutes (′)	-	1 hour *h.*
24 hours	-	1 day.
7 days	-	1 week.
52 weeks 1 day, 6 hours; or 365 days 6 hours; or 12 months, make		1 year.

235. Note—The 6 hours in each year are not reckoned till they amount to one day: hence, a common year consists of 365 days,and every fourth year, (called leap year or bissextile,) of 366 days. That day is added to February, which then has 29 days. The number of days in each month is thus found:

236. Thirty days has September,
April, June and November;
February twenty-eight alone;
And all the rest have thirty-one.

or as follows:

237. The fourth, eleventh, ninth and sixth,
Have thirty days to each affixed:
And every other thirty-one,
Except the second month alone,
Which has but twenty-eight, in fine,
Till leap year gives it twenty-nine.

238. When the number, representing the year of our Lord, can be divided by 4 without a remainder, it is bissextile or leap year, in which February has 29 days—for instance: the year 1820 being divisible by 4 exactly, is a leap year; but is is not so of the year 1821, which leaves a remainder.

1. Reduce 6 years to months. 72.
2. Bring 9 years to weeks, at 52 weeks per year. 468.
3. Bring 7 days to hours. 168.
4. Bring 12 hours to minutes. 720.

* 1 month is 4 weeks, or 30 days—not exactly however.

5. Bring 28 days to hours. 672.

6. How many days from June 6th included, to October 9th included? 126.

7. Was the year 1784 leap year?

8. How many hours has a boy 11 years old lived, allowing 365 days 6 hours to each year? 96426.

9. Reduce 56 minutes to seconds. 3360.

10. Reduce 4 weeks to minutes. 40320.

11. 1 day is what part of 1 week?

12. In 14 weeks how many days? 98.

13. 1 hour is what part of 1 day?

14. 1 second is what part of 1 minute? $\frac{1}{60}$.

15. How many days from the 8th of 2d month 1821, to 17th of 7th month? 160d.

16. How many days from 1769 to 1785, each common year being 365 and each bissextile 366 days? 5844.

17. How many days from the commencement of the war between England and America—April 19th 1775, to the peace of January 20th 1783? 2834 days.

18. Bring 6 days 20 hours and 18 minutes to seconds. 591480.

19. Bring the month of May or 5th month to hours and seconds. 744 hrs. 2,678,400″.

20. Reduce 4 weeks and 3 days to seconds. 2,678,400.

21. How many days from the 20th of January or 1st month, to 7th September or 9th month of the year 1816? 232.

239. MOTION OR CIRCLE MEASURE.

1 circle, (360 degrees) equal 12 signs.
1 si. - 30 degrees.
1 ° - 60 minutes.
1 ′ - 60 seconds (″).

60 seconds (″) make 1 minute (′).
60 ′ - 1 degree (°)
30 ° - 1 sign. s.
12 s. or 360° - 1 circle.

This is used by astronomers, navigators, &c.

1. Bring 6 signs to degrees. 180.

2. Bring 7 degrees to minutes. 420.

3. Bring 1 circle to minutes. 21600.

4. 1 degree is what part of 1 circle? $\frac{1}{360}$.

5. 1 sign is what part of 1 circle? $\frac{1}{12}$.

6. When the sun has reached the 1st degree of the 8th sign, how many degrees has it gone through? 211 deg.

7. Reduce 3 signs 18° and 6′ to minutes. 6486.

8. Bring 27° 54′ to seconds. 100440.

240. 12 particular things make 1 dozen.
12 doz. - - 1 gross.
12 gross, or 144 doz. 1 great gross.

ASCENDING REDUCTION.

241. WHEN numbers are to be changed to higher denominations, the reduction is *ascending*.

This is, likewise, nothing but a single rule of three. For, when I wish, for instance, to reduce 146 shillings to pounds, knowing that 1*l.* makes 20*s.* I have to state as follows: *xl.* : 1*l.* :: 146*s.* : 20*s.* and it is easy to see, that to find the 1st term, I have merely to divide 146 by 20; because, the multiplication by one, needs not be performed. Hence we draw this

1ST GENERAL RULE.

242. Divide the given quantity by the number which makes one of the next higher denomination; the quotient will show how many there are of that higher denomination, and the remainder, if any, will be of the same denomination as the quantity divided.

In the above example, 146, divided by 20 (number of shill. in 1 pound) gives 7 for quotient, and 6 for remainder—or 7*l.* 6*s.*

FEDERAL MONEY.

243. To reduce mills to dollars, divide by 1,000, or move the dot 3 classes to the left.

To reduce cents to dollars, divide by 100, or move the dot 2 classes to the left.

To reduce dimes to dollars, divide by 10, or move the dot 1 class to the left.

This is founded on what has already been demonstrated, (art. 22.)

1. Bring 17000 mills to drs. $17.
2. " 6300 " $6.30.
3. " 630 cents to drs. 6.30.
4. " 1820 cents to drs. 18.20.
5. In 780 dimes, how many drs.? $78.
6. " 421 dimes " 42.10 cents.
7. " 1262 cents " 12.62.
8. " 50 mills "

NOTE 1.—To change fourths of a cent to cents, divide them by 4.—To change thirds of a cent to cents, divide by 3.—Halves of a cent, divide by 2.

9. Bring 24 fourths of a cent to cents. 4)24(6 cents.
0

10. " 34 fourths of a cent to cents. 8½ cts.
11. " 27 thirds " 9 cts.
12. " 740 " " and then to drs. 2.46⅔ cts.
13. " 14 halves of a cent to cents. 7 cts.
14. " 75 halves of a cent to cents. 37½ cts.

ENGLISH MONEY.

4 farthings or quarters	make	1 penny.
12 pence	"	1 shilling.
20 shillings	"	1 pound.

NOTE 2.—To change shillings to pounds, divide them by 20; because 20 shillings make 1 pound.

1. Bring 80 shillings to pounds. 2,0)8,0(4 pounds.
2. " 124 shillings to pounds. 2,0(12,4(6*l.* 4*s.*
3. " 137 shillings to pounds. 2,0)13,7(6*l.* 17*s.* after taking the half or dividing by 2, count the remainder 1, as 10, and add it to the 7.
4. Bring 354 shillings to pounds. 17*l.* 14*s.*
5. " 299 shillings to pounds. 14*l.* 19*s.*
6. " 500 shillings to pounds. 15*l.*

NOTE 3.—To bring pence to shillings, divide them by 12, because 12 pence make 1 shilling.

7. Bring 96 pence to shillings. 12)96(8 shillings.
8. " 57 pence to shillings. 12)57(4*s.* + 9*d.*
9. " 144 pence to shillings. 12*s.*
10. " 600 pence to shillings. 50*s.*

NOTE 4.—To bring farthings to pence, divide them by 4, because 4 farthings make 1 penny.

11. Change 32 farthings to pence. 4)32(8*d.*
12. " 45 " " 4)45(11*d.* + 1 *qr.*
13. " 723 " " 18*d.* + 3 *qrs.*

2D GENERAL RULE.

244. When there are several denominations, change the lower to the next higher, this to the next again, &c.—The successive remainders must be of the same denomination as the quantity divided, and the last quotient will be the highest denomination.

NOTE 5.—To change pence to pounds, bring them first to the next higher denomination, viz: shillings; and then, those shillings to pounds.

14. Bring 650 pence to pounds.

12)650	I first reduce the pence to shillings by dividing by 12, and find 54 shillings, and 2 for remainder; these are 2 pence. I next divide the 54 shillings by 20, to reduce them to pounds, and find 2 pounds, and a remainder of 14, which I call 14s. The result is then 2*l.* 14*s.* 2*d.*
2,0) 5,4*s.* + 2*d.*	
2*l.* 14*s.*	
2*l.* 14*s.* 2*d.*	

15. Bring 2328 pence to shillings and pounds. 9*l.* 14*s.*
16. See if 1920*d.* equal 8 pounds.
17. Bring 6004 pence to pounds. 25*l.* 0*s.* 4*d.*
18. Change 2694 pence to pounds. 11*l.* 4*s.* 6*d.*

NOTE 6.—To reduce farthings to pounds, bring them first to pence, then those pence to shillings, and then bring those shillings to pounds.

19. Reduce 2478 farthings to pence, shillings, and pounds.

4)2478

12)619*d.* + 2 *qrs.*

2,0)5,1*s.* + 7*d.*

2*l.* 11*s.*

Result.
2*l.* 11*s.* 7*d.* 2*qrs.*

20. How many pounds in 7408 farthings? 7*l.* 14*s.* 4*d.*
21. See if 7680 farthings make 8 or 9 pounds.
22. Bring 10752 farthings to pounds. 11*l.* 4*s.*
23. Bring 963 *qrs.* to pounds. 1*l.* 0*s.* $0\frac{3}{4}$*d.*
24. Are 144 farthings worth 3 or 4 shillings?
25. Reduce 95999 farthings to pounds. 19*l.* 19*s.* $11\frac{3}{4}$*d.*

AVOIRDUPOIS WEIGHT.

16 drams make		1 ounce, *oz.*
16 oz.	-	1 pound, *lb.*
28 lb.	-	1 quarter, *qr.*
4 qrs.	-	1 hundred weight, *cwt.*
20 cwt.	-	1 ton, *T.*

1. Reduce 480 cwt. to tons. 24 tons.
2. Bring 76 qrs. to cwt. 19 cwt.

3.	Are 112 oz. equal to 7 ℔.?	
4.	Reduce 258 drams to ounces.	16 oz. 2 dr.
5.	" 170 ℔. to qrs.	6 qrs. 2 ℔.
6.	" 2240 ℔. to tons.	1 *T.*
7.	" 1792 oz. to cwt.	1 cwt.
8.	" 26848 dr. to qrs.	3 qrs. 20 ℔. 14 oz.
9.	" 35845 oz. to tons.	1 *T.* and 5 oz.

TROY WEIGHT.

24 grains (gr.)	make	1 pennyweight, *dwt.*
20 dwt.	-	1 ounce, *oz.*
12 oz.	-	1 pound, *lb.*

1.	Reduce 96 oz. to pounds.	8 ℔.
2.	" 214 oz. to pounds.	17 ℔. 10 oz.
3.	" 187 dwts. to ounces.	9 oz. 7 dwts.
4.	" 5760 grains to dwts. and to ℔.	1 ℔.
5.	" 486 grains to oz.	1 oz. and 6 gr.
6.	" 145 dwts. to oz.	7 oz. 5 dwts.
7.	" 963 dwts. to ℔.	4 ℔. and 3 dwts.

APOTHECARY WEIGHT.

20 grains (gr.)	make	1 scruple, ℈.
3 ℈	"	1 dram, ʒ.
8 ʒ	"	1 ounce, ℥.
12 ℥	"	1 pound, ℔.

1.	Reduce 48 ounces to pounds.	4 ℔.
2.	" 338 drams to ounces.	42 ℥ 2 ʒ.
3.	" 38 scruples to drams.	12 ʒ 2 ℈.
4.	" 185 grains to scruples.	9 ℈ 5 gr.
5.	" 7498 drams to ℔.	78 ℔ 10 ℥.
6.	" 288 scruples to ℔.	1 ℔.
7.	" 5760 grains to ℔.	1 ℔.
8.	" 25 scruples to ounces.	1 ℥ 0 ʒ 1 ℈.
9.	" 2115 scruples to ℔.	7 ℔ 4 ℥ 1 ʒ.

LONG MEASURE.

3 barley corns	make	1 inch, *in.*
12 in.	"	1 foot, *ft.*
3 ft.	"	1 yard, *yd.*
5½ yds.	"	1 pole or perch, *per.*
40 per.	"	1 furlong, *fur.*
8 fur.	"	1 statute mile, *st. m.*
69½ st. miles, or 60 geographic miles	}	1 degree, *deg.*
360 degrees,		1 circle, *cir.*

NOTE.—3 miles make one league—6 feet make 1 fathom—4 inches make 1 hand.

245. To reduce yards to poles, and to reduce statute miles to degrees, divide by $5\frac{1}{2}$ and $69\frac{1}{2}$. Pupils are generally unable to perform these divisions; because, having in the assistants no directions how to proceed, they cannot *guess*, how to divide by the $\frac{1}{2}$. Let them be informed, then, that whenever there is a fraction as $\frac{1}{2}$, or $\frac{1}{3}$, or $\frac{1}{4}$, or any other, in the divisor, he must, previous to the division, reduce that divisor into halves, thirds, fourths, &c. &c. and likewise reduce the dividend into the same, so as not to alter the value of the quotient. The remainder, if any, must be considered as halves, thirds, fourths, &c.

Ex. Reduce 265,760 yds. to degrees.

```
5½)265760 yds.
 2       2
——  ———————
11  )531520
————————————
  4,0) 4832,0 perches.
       ———————
    8) 1208 fur.
       ————
  69½) 151 miles.
   2     2
 ————  ————
 139   )302(2 deg. 12 miles.
        278
        ———
         24 halves=12 wholes.
        ———
```

In order to divide by $5\frac{1}{2}$, I reduce that divisor to halves, and obtain 11; I likewise multiply the dividend by 2, so as not to change the quotient. The $69\frac{1}{2}$ statute miles must also be reduced to halves, as well as the dividend 151. The remainder 24, are evidently 24 halves, equal to 12 whole miles.

1. Reduce 180 geographic miles to degrees. 3 deg.
2. Reduce 1251 statute miles to degrees. 18 deg.
3. Bring 280 perches to furlongs. 7 fur.
4. Are 556 furlongs equal to one degree?
5. Are 5280 feet equal to 1 mile?
6. Bring 31680 inches to furlongs. 4 fur.
7. Bring 66 feet to perches. 4 per.
8. Bring $16\frac{1}{2}$ feet to perches. 1 per.
9. Bring 43200 geographical miles to circles. 2 cir.
10. In 7040 yards how many miles? 4 miles.
11. In 4620 yards how many miles? 2 miles 5 fur.

12. In 52 feet how many yards?	17 yds. 1 foot.
13. In 2880 poles how many leagues?	3 lea.

CLOTH MEASURE.

$2\frac{1}{4}$ inches make 1 nail, *na*.*
4 na. - 1 quarter of a yd. *qr*.
4 qrs. - 1 yard, *yd*.

NOTE.—3 qrs. of an Amer. yd. make 1 ell Flemish, *E. F.*
5 - - - 1 ell English, *E. E.*
6 - - - 1 ell French, *E. Fr.*

1. Bring 40 quarters to yards.	10 yds.
2. " 12 nails to quarters.	3 qrs.
3. " 27 inches to quarters.	3 qrs.
4. " 135 quarters to English ells.	27 E. E.
5. " 162 inches to E. Fr.	3 E. Fr.
6. " 248 inches to yds.	6 yds. 3qrs. 2 na. $\frac{1}{2}$ in.
7. " 56 nails to yds.	3 yds 2 qrs.
8. " 95 nails to ells English.	4 E. E. 3 qrs. 3 na.
9. " 1467 inches to ells Flemish.	27 E. F. 1 qr.

LAND MEASURE.

144 square inches (sq. in.) make 1 square foot.
9 sq. ft. - - - 1 sq. yard
$30\frac{1}{4}$ sq. yds.† - - - 1 sq. perch
40 sq. per. - - - 1 sq. rood
4 sq. r. - - - - 1 sq. mile

1. Bring 2,574 sq. acres to sq. miles.	4 m. 14 a.
2. " 1,120 sq. per. to sq. acres.	7 a.
3. " 58,080 sq. yds. to sq. acres.	12 a.
4. " 164,592 sq. in. to sq. per.‡	4 per. 6 yds.
5. " 16,940 sq. yds. to sq. acres.	3 a. 2 r.
6. " 1,296 sq. in. to sq. yds.	1 yd.
7. " 360 sq. per. to sq. r.	9 r.
8. " 40,356 sq. feet to sq. r.‡	3 r. 28 per. 7 yds.
9. " 1,344,168 sq. in. to sq. per.§	34 per. 8 yds. 6 ft.

* See art. 245.
† See the same art.
‡ Attend to the remainders.
§ This example cannot be solved by ascending reduction. See the manner of doing it, (compound division, art. 267) which way is always the surest, when the subdivisions of the integers contain fractions, as 1-2, 1-3, 1-4, &c.

7.92 In.	= 1 link.	16 sq. poles	= 1 sq. chain.
25 links	= 1 pole.	10 sq. chains	= 1 acre sq.
4 poles or 100 links	= 1 chain.		

1. Reduce 750 links to chains. 7 ch. 2 poles.
2. " 75 links to poles. 3 poles.
3. " 7,128 inches to chains. 9 ch.
4. " 1,184 sq. poles to sq. chains. 74 sq. chains.
5. " 1,232 sq. poles to sq. acres. 7 sq. a. 7 ch.
6. " 3,260 links to chains. 32 chains 2 p. 10 links.

CUBIC OR SOLID MEASURE.

1728 solid inches	make	1 solid foot.
27 sol. ft.	- - - - -	1 sol. yard.
40 feet round timber, 50 feet hewn timber,	- -	1 ton or load.
128 solid feet, viz. 8 feet in length, 4 in breadth, and 4 in heighth,		1 cord of wood.

246. NOTE.—To calculate the contents of a solid body, multiply its length by its breadth, and this product by the thickness or height.

1. Thus, to find the solid contents of a room 12 feet broad, 15 long, and 11 high; multiply 12 by 15 = 180, and this product by the height 11, which give 1980 solid feet, for the solid contents of the room. If now, I wish to know how many cubic *yards* that room contains; since it takes 27 solid feet to make 1 solid yard, I divide 1980 feet, by 27, and find 73 yards, and 9 feet over.

2. In a pile of wood 96 feet long, 5 feet high, and 4 wide, how many cords? 15.

3. What are the contents of a load of wood 5 feet long, 5 broad, and 5 high? 125 ft.

4. Bring 512 feet to cords. 4 cords.

5. " 694 feet to cords. 5 cords 2 yds.

6. Is a pile of wood 7 feet long, 6 broad, and 3 feet high, equal to 1 cord? Not by 2 feet.

7. How many cords of wood can I put in a cellar 17 feet long, 14 wide, and 9 high? 16 cords and 94 solid feet.

LIQUID MEASURE.

4 gills (gi.)	make	1 pint.
2 pts.	- - -	1 quart.
4 qrts.	- - -	1 gallon.
63 gall.	- - -	1 hogshead.
2 hhds.	- - -	1 pipe or butt.
2 pipes or butts	-	1 tun, *T*.

NOTE 1.—In Pennsylvania, 16 gall. make a ½ barrel—31½ gall. 1 barrel—42 gall. 1 tierce—64 gall. 1 double barrel—84 gall. 1 puncheon.

NOTE 2.—In the eastern states, 10 gall. make an anchor of brandy, (anc.) 18 gall. 1 runlet, (run.)

1.	Bring 10 pipes to tuns.	5 tuns.
2.	" 786 gallons to hhds.	12 hhds. 30 gall.
3.	" 163 quarts to gallons.	40 gal. 3 qrts.
4.	" 2016 pints to tuns.	1 T.
5.	" 384 pints to half barrels.	3 half bar.
6.	" 288 gills to gallons.	9 gall.
7.	" 3584 pints to double barrels.	7 doub. bar.
8.	" 524 pints to anchors.	6 anc. 5 gall. 2 qrts.
9.	In 1888 qrts. how many pipes, tierces, and anchors?	3 pipes, 2 tier. 1 anc.

DRY MEASURE.

2 pints (pts.)	make	1 quart.
8 qrts.	-	1 peck.
4 pe.	-	1 bushel, *bu.*

1.	Bring 72 pecks to bushels.	18 bu.
2.	" 112 quarts to pecks.	14 pe.
3.	" 19 pints to quarts.	9 qts. 1 pt.
4.	" 96 pints to pecks.	6 pe.
5.	" 64 pints to bushels.	1 bu.
6.	" 47 pecks to bushels.	11 bu. 3 pe.
7.	" 18 pints to half pecks.	2 half pe. 1 qrt.
8.	" 518 pints to bushels.	8 bu. 3 qrts.

NEW ENGLAND DENOMINATIONS.

2 pints	make	1 quart.	2 bu.	make	1 strike.
2 qts.	-	1 pottle.	2 str.	-	1 coom.
2 pot.	-	1 gallon.	2 co.	-	1 quarter.
2 gall.	-	1 peck.	5 qrs.	-	1 wey.
4 pecks	-	1 bushel.	2 weys	-	1 last.

4 quarters make 1 chaldron.

1.	Bring 32 quarters to lasts.	3 lasts 2 qrs.
2.	" 24 bushels to quarters.	3 qrs.
3.	" 488 gallons to bushels.	61 bu.
4.	" 125 pottles to pecks.	31 pks. 1 pot.
5.	" 112 pints to gallons.	14 gal.
6.	" 640 gallons to lasts.	1 last.
7.	" 192 bushels to chaldrons.	6 ch.

8. Bring 2,056 gallons to lasts. 3 lasts, 2 qts. 1 bu.
9. " 124 pints to pecks. 7 pe. 6 qts.

TIME.

60 seconds (sec. or ″)	make	1 minute.
60 min. (′)	- - - - -	1 hour.
24 h.	- - - - - - - -	1 day.
7 d.	- - - - - - - -	1 week.
52 weeks, 1 day, 6 hrs.; or 365 days 6 hrs.; or 12 months, make		1 year.

1. Bring 72 months to years. 6 years.
2. " 468 weeks to years, at the rate of 52 weeks per year. 9 years.
3. Bring 168 days to weeks. 24 w.
4. " 40,320 minutes to weeks. 4 w.
5. " 591,480 seconds to days. 6 d. 20 hrs. 18 min.
6. " 2,678,400 seconds to weeks. 4 w. 3 d.

MOTION, OR CIRCLE MEASURE.

60 seconds (″)	make	1 minute, (′)
60 minutes	- -	1 degree, (°)
30°	- - - - -	1 sign, (*sig.*)
12 sig. or 360°	- -	1 circle.

1. Bring 180° to signs. 6 signs.
2. " 420 minutes to degrees. 7 degrees.
3. " 21,600 ′ to circles. 1 circle.
4. In 211 degrees how many signs? 8 sig. 1 deg.
5. Reduce 6,486 ′ to signs. 3 sig. 18° 6′.
6. " 100,440 ″ to degrees. 27° 54′.

247. Grindstones are sold by the the cubic foot, commonly called a stone, and the contents are found by the following

RULE.

To the whole diameter add half of the diameter; multiply their sum by the same half, and this product by the thickness.

As the dimensions are generally given in inches, you must divide this product by 1728, the number of inches in a cubic foot, and the quotient will give the answer.

Ex. 1. How many cubic feet in a grindstone, 34 inches in diameter, and 4 inches thick?

```
              34 in. diameter,
              17 in. half diameter.
              --
              51
Multiply by   17 the half diameter.
             ---
             357
             51
             ---
             867
Multiply by    4 the thickness.
            ----
   1728)3468(2 solid feet+1/144.
        3456
        ----
          12
```

2. What are the contents of a grindstone 36 inches diameter and 4 inches thick ? $2\frac{1}{4}$ cub. ft.

3. How many cubic feet in a grindstone 24 inches diameter and 4 inches thick ? 1 foot.

Note 1.—When required to divide money of one denomination into several coins of different values, but each in equal number :

1st. Reduce each sort of coin into the lowest denomination mentioned, and add them together for a divisor.

2d. Reduce the given money to the same denomination, for a dividend.

3d. Divide, and the quotient, will be the required number.

Note 2.—Observe the same directions for weights and measures.

504 dollars are to be divided between a mother, her son and her daughter, so that for every quarter of a dollar given to the daughter, the boy is to receive half a dollar, and the mother one dollar; how much will each receive ?

Here the lowest denomination mentioned being quarters, I say,

```
1 dollar=4 quarters.        $ 504
½   "    2    "                4
¼   "    1    "             -----
         -    "             7)2016 Dividend.
Divisor 7 quarters.         -----
                              288 quotient or required
```

number—i. e. 288 dollars for the mother—288 halves for the son, and 288 quarters for the daughter.

How many crowns of 5*s*. each, half crowns, and shillings, are in equal number, in 279*l*. 13*s*.? 658.

PROMISCUOUS QUESTIONS.

How many dimes are there in 60 dollars?
How many shillings are there in 11 pounds?
How many pence in 220 shillings?
How many quarters of a cent in 10 cents? in 15? in 18?
In 72 pence how many shillings? How many in 204?
How many pence in 144 shillings?
Bring 72 half pence to shillings.
How many pounds in 2 tons? in 4? in 7?
How many pounds in 64 ounces (avoir. weight)? in 128?
How many ounces in 24 pounds?
How many scruples in 15 drams? in 8 drams? in 12? in 9?
How many drams in 60 scruples? in 21? in 45? in 93?
How many pennyweights in 96 grains (troy)? in 72?
Bring 5 pennyweights to ounces?
How many feet in 1 mile? in 2? in 8? in 9? in 11?
How many yards in 36 feet? in 42? in 96? in 38? in 74?
How many inches in 1 foot? in 7? in 12? in 11? in 9?
How many half inches in 10 feet? in 4? in 25?
How many rods in 33 yards? in 44? in 55? in 99? in 102?
How many yards in 8 quarters? in 11? in 12? in 23?
How many inches in 4 half yards? in 6?
How many inches in 1 quarter of a yard? in 3? in 9?
How many quarters in 16 yards? in 30?
How many hours in 1 week? in 3? in 7? in 52?
How many pecks in 1 bushel? in 4? in 6? in 15?
How many bushels in 38 pecks? in 40? in 90? in 72?
How many quarts in 1 bushel? in 5? in 9? in 12?

1. In 15 pieces of cloth, each piece 20 yards; how many French ells? 200.

2. How many lockets, each to weigh half an ounce, will 4560 grains of gold make? 19.

3. If 22 acres, 2 roods, are sold at the rate of 17*s*. per rood, how many six-pences, will it take to pay for them? 3060 six-pences.

4. If I buy 5 hhds. and $\frac{1}{2}$ a barrel of whiskey at the rate of 1 dollar for 6 gallons; how much must I pay? \$55.16$\frac{2}{3}$.

5. How many cups, each of 11 oz. will 19 lb. 3 oz. silver make? 21.

6. A teacher in Boston, asks his pupil, how many pounds, dollars and shillings, of each an equal number, there are in 54 guineas, each worth 28 shillings? 56.

7. What would be the answer in Pennsylvania, where the dollar is worth *7s. 6d.* and the guinea 35*s.*? 66, and 9s. over.

8. If a druggist sell 7 ℔ 3 ℥ 4 ʒ vermillion at 8*d.* a dram; what will be the amount in pounds, shillings, &c.? 23*l.* 6*s.* 8*d.*

9. If a tavern keeper sell half a barrel of beer at the rate of $6\frac{1}{4}$ cents a pint; how many dollars will he receive? $8.

10. And likewise a barrel of brandy at $6\frac{1}{4}$ cents per gill; what will he receive? and what will he clear supposing he gave $1.50 per gallon.? He will clear $15.75.

11. In 17 cwt. 1 qr. 6 lb. how many parcels, each of 34 pounds? 57.

12. Required the number of turns a wheel 18 feet 4 in. in circumference will make, in running 150 miles? 43,200.

13. Imported from Rotterdam, 46 bales of linen, each containing 24 pieces, and each piece 42 ells Flemish: how many yards is it? 34,776 yds.

14. How many rings each weighing 5 dwt. 7 gr. may be made of 3 lb. 5 oz. 16 dwt. 2 gr. of gold? 158.

15. How many dozen of gallons, quarts and pints, of each a like number, will be required, to draw off a cask of wine containing 165 gallons? 10 doz.

16. If a ship's cargo be 250 pipes, 130 hogsheads, and 150 half ditto; how many gallons in all? And allowing every pint to be a pound, what burden was the ship of?

44,415 gal. 158 T. 12 C. 2 qrs.

17. In 86,400 turns of a wheel 18 ft. 4 in. in circumference, how many miles? 300.

COMPOUND ADDITION.

248. To perform this operation, place the same denominations under each other, in different columns; add the lowest denomination, set the sum under, if less than 1 of the next higher; but if more, reduce it to the higher denomination, and merely set down the overplus or remainder, if any, or 0, if nothing remain; and carry those found, to the next column, to be added to those it contains; and so on, till you come to the integers which are added as usual.

EXAMPLE.

1.	4*l.*	3*s.*	2*d.*
	16	14	$7\frac{1}{4}$
	24	18	$11\frac{1}{4}$
	75	15	$4\frac{1}{4}$

After placing pounds under pounds, shillings under shillings, pence under pence, and farthings under farthings, I begin by addding the lowest denomination viz. the farthings; and as there are but 3 farthings, a sum not equal to one penny, I set down 3 under the column of farthings. Finding 24 pence under the 2d column, I convert them into shillings, by dividing by 12; and as there are 2 shillings and no overplus, I set down 0 under that row, and adding the 2 shillings to the column of shillings, I find 52 shillings; which, being reduced to pounds, give 2 pounds and 12 shillings; I set down 12 shillings, and adding the 2 pounds to those already given, I find 121. So that the whole sum is 121*l.* 12*s*, $0\frac{3}{4}$*d.*

2. Add 7*l.* 2*s.* 4*d.*—33*l.* 19*s.* $9\frac{3}{4}$*d.*—88*l.* 18*s.* 8*d.*—77*l.* 17*s.* $7\frac{3}{4}$*d.*

3. Sum up 10*l.* 10*s.* $10\frac{3}{4}$*d.*—78*l.*—0*l.* 6*s.*—0*s.* $2\frac{1}{4}$*d.*—147*l.* 2*d.*

4. Add 27*l.* 9*s.* $6\frac{1}{4}$*d.*—183*l.* 18*s.* $11\frac{1}{2}$*d.*—19*s.* $\frac{3}{4}$*d.*—19*l.* 19*s.* $11\frac{3}{4}$*d.*

5. Bought 1 watch for 3*l.* 14*s.* 9*d.*—a seal, 2*l.* 17*s.* 6*d.*—a key, 1*l.* 11*s.* 6*d.*—3 rings, each 9*s.* $10\frac{1}{2}$*d.*—how much must I pay for the whole? Ans. 9*l.* 13*s.* $4\frac{1}{2}$*d.*

6. The price of a side-board being 32*l.* 15*s.*—of a carpet 13*l.* 4*s.* $9\frac{1}{4}$*d.*—of a looking glass 7*l.* 7*s.* $7\frac{1}{2}$*d.*—of 2 tables, each 4*l.* 18*s.* 10*d.*—of 6 chairs altogether 8*l.* 14*s.* $11\frac{1}{4}$*d.*—what is the value of the whole? 72*l.* 0*s.* $\frac{1}{2}$*d.*

7. A man owes his baker seventeen pounds, six-pence and two farthings; to his grocer, thirty-eight pounds, two shillings and three farthings—to his butcher twenty-nine pounds five shillings—to his milk man 142 shillings, ten-pence and two farthings; what is the amount of the whole? 91*l.* 10*s.* $0\frac{1}{4}$*d.*

TROY WEIGHT.

	lb.	*oz.*		*lb.*	*oz.*	*dwt.*		*lb.*	*oz.*	*dwt.*	*gr.*
1.	3	7	2.	16	10	18	3.	104	11	16	8
	26	4		29	7	12		0	2	14	22
	80	11		37	3	19		0	9	19	17
	13	9									
	124	7									

4. What is the sum of forty-two pounds and twenty grains —nine ounces, fifteen pennyweights—three pounds sixteen pennyweights and twenty one grains—six ounces and eleven grains? 46 lb. 4 oz. 13 dwt. 4 gr.

5. Bought several plates weighing 19 lb. 10 oz. 16 dwt.— tumblers, weighing 3 lb. 9 oz. 12 dwt. 17 gr.—spoons weighing 2 lb. 11 oz. 9 dwt. 20 gr.—forks 2 lb. 0 oz. 18 dwt. 16 gr. —and a cream cup, weight 11 oz. 0 dwt. 10 gr.—what is the weight of the whole? 29 lb. 7 oz. 17 dwt. 15 gr.

AVOIRDUPOIS WEIGHT.

	T.	*cwt.*	*qrs.*
1.	50	14	2
	72	18	3
	1	7	1
	16	19	2
	142	0	0

	T.	*cwt.*	*qrs.*	*lb.*
2.	5	0	1	18
	27	18	0	26
	10	7	3	11
	7	19	2	20

	cwt.	*qrs.*	*lb.*	*oz.*	*dr.*
3.	12	2	18	14	12
	7	0	22	10	7
	18	3	4	15	15
	19	3	27	11	9

4. Add 16 tons, 15 cwt. and 6 pounds—18 cwt. 3 qrs. and 20 pounds—15 tons 1 qr. 13 lb.—3 tons 14 cwt. 12 lb.
Ans. 36 tons 8 cwt. 1 qr. 23 lb.

5. Add 3 qrs. 21 lb. 11 oz.—17 cwt. 17 lb. 14 oz and 8 dr. —12 tons 19 cwt. 1 qr. 8 dr.—13 tons, 15 cwt. 24 lb. 10 oz. 15 dr.—6 lb. 7 oz. 8 dr.—3 qrs. 7 oz. 12 dr.
Ans. 27 T. 13 cwt. 1 qr. 15 lb. 4 oz. 3 dr.

6. A man bought 4 lots of hay; the 1st weighed 5 tons, 12 cwt. 1 qr. 16 lb.—the 2d weighed 1 ton, 17 cwt. 3 qrs. 26 lb.—the 3d weighed 19 cwt. and 12 lb.—and the last 18 tons 3 qrs.—What was the weight of the whole?
26 tons 10 cwt. 0 qr. 26 lb.

7. What quantity of hops is there in 5 bags, the two first weighing each 1 qr. and 25 lb. and each of the others 8 lb. 8 oz. more? 2 cwt. 0 qr. 5 lb. 0 oz.

APOTHECARIES WEIGHT.

	℔	℥	ʒ
1.	6	10	5
	8	7	6
	14	8	2
	15	11	7

	℔	℥	ʒ	℈
2.	0	9	3	1
		7	7	0
	1	10	6	2
	2	6	5	1

	℔	℥	ʒ	℈	*gr.*
3.	2	11	2	0	16
			7	2	14
		9	4	1	8
	1	7	2	2	12

4. Add 3 pounds and 6 drams—8 ounces, 4 drams, and 2

scruples—7 pounds, 2 ounces, 5 drams—9 ounces, 7 drams, and 17 grains—3 drams, 2 scruples, 18 grains.

10℔ 10℥ 2ʒ 2℈ 15 gr.

5. Add 3 lb. 3 ounces—2 pounds, 5 grains—1 ounce, 6 drams, 2 scruples, 12 grains—6 drams, 6 grains—3 ounces, 12 grains.

5℔ 8℥ 5ʒ 0℈ 15 gr.

LONG MEASURE.

	Deg.	*G.m.*
1.—	2	46
	6	32
	1	56
	16	37

	L.	*M.*	*F.*	*P.*
2.—	8	1	6	27
	0	2	4	34
	5	0	5	5
	4	1	3	20

	yd.	*ft.*	*in.*	*bc.*
3.—	7	2	10	1
	2	2	8	2
	3	1	11	0
	8	0	9	1

4. Add 7 degrees, 14 geographic miles—1 degree, 50 geographic miles—42 geog. miles—3 degrees, 17 geog. miles.

13 deg. 3 geog. miles.

5. Add 16 leagues, 1 mile, 4 furlongs—2 miles, 17 poles—14 leagues, 2 miles—7 furlongs, 31 poles—2 leagues, 36 poles—1 mile, 17 poles—3 leagues, 5 furlongs.

37 lea. 2 m. 2 fur. 21 poles.

6. A waggon travels 32 miles, 3 fur. in one day—the next day, 40 miles and 28 poles—the two following days, each, 36 miles, 5 fur. 10 ps. How far will it be from the place of departure?

7. Add 3 yds. 2 feet, 7 inches—2 feet, 10 inches, and 1 barl. corn—7 yds. 9 inches—8 inches, 2 barl. corns—8 yds.—1 foot, 3 inches—1 yd. 11 inches, 2 barl. corns.

22 yds. 0 f. 1 in. 2 bar.

CLOTH MEASURE.

	Yd.	*qr.*	*na.*
1.—	6	3	2
	17	1	1
	0	3	2
	14	1	3

	E.E.	*qr.*	*na.*
2.—	72	4	2
	8	3	3
	14	2	0
	19	0	1

	E. Fr.	*qr.*	*na.*
3.—	16	4	2
	7	5	1
	92	2	3
	28	5	0

	E. F.	*qr.*	*na.*
4.—	77	2	3
	11	1	2
	35	2	0
	48	0	1

5. Add 12 yards 2 nails—7 yds. 3 qrs. 3 na.—19 yds. 1 qr. 2 na.—3 qrs. 1 na.—102 yds. 3 na.—153 yds. 1 qr. 1 na.

295 yds. 3 qrs.

6. Add 13 Fl. ells 3 na.—27 Fl. ells 2 qrs. 1 na.—12 Fl. ells 1 qr. 2 na. 53 Fl. E. 0 qr. 2 na.

7. Add 6 E. ells 4 qr. 3 na.—18 E. ells 2 na.—2 qr. 2 na. —72 E. ells 1 qr. 97 E. E. 3 qrs. 3 na.

8. A merchant receives 5 pieces of French cloth; No. 1 contains 38 Fr. ells 3 qrs.—No. 2, 43 Fr. ells 5 qrs.—No. 3, 41 Fr. ells 2 qrs. 2 na.—Nos. 4 and 5, each 45 Fr. ells 5 qrs. how much in all? 215 Fr. E. 2 qrs. 2 na.

LAND MEASURE.

	A.	*R.*	*P.*		*yd.*	*ft.*	*in.*		*A.*	*R.*	*P.*
1.—	12	3	27	2.—	46	8	72	3.—	28	0	39
	18	1	36		70	3	48		20	7	15
	14	0	20		12	6	110		35	6	32
	22	2	18		25	5	88		31	4	20

4. Add 25 acres, 2 roods, 30 perches—401 acres, 3 roods 15 per.—89 acres 38 perches—17 roods, 17 perches—609 acres, 1 rood, 34 perches. 1130 A. 2 R. 14 per.

5. Add 23 yds. 4 ft. 100 inches—67 yds. 7 ft.—12 yds. 82 in.—6 ft. 38 in. 104 yds. 0 ft. 76 in.

6. A man owns 4 lots of ground; Nos. 1 and 2, each contains 18 acres, 1 rood, 28 perches—Nos. 3 and 4, each 1 acre, 3 roods, 38 perches. How much in all? 40 A. 3 r. 12 p.

CUBIC, OR SOLID MEASURE.

	Cords.	*ft.*	*in.*		*yds.*	*ft.*	*in.*
1.—	2	100	608	2.—	14	22	1518
	12	64	864		18	14	1700
	6	96	1292		4	26	1725
	17	125	1684		72	10	444

3. Add up 16 cords, 75 feet—21 cords, 94 feet, 1240 inches—16 cords, 1152 inches—18 cords, 98 feet, 1246 inches—48 feet, 1710 inches. 73 cords, 62 ft. 164 in.

4. Add 41 yards, 21 feet, 1206 inches—6 yds. 12 feet—16 feet, 1500 in.—12 yds—1714 inches. 49 yds. 9 ft. 964 in.

5. In a lot of wood there are 42 cords, 114 feet, 678 inches—in another, 17 cords, 28 feet, 1612 inches; in another, 34 cords, 110 feet, 1212 inches. How much in all?

94 cds. 126 ft. 46 in.

LIQUID MEASURE.

	T.	hhd.	gal.		hhd.	gal.	qt.		gal.	qt.	pt.
1.—	6	2	46	2.—	12	60	3	3.—	18	3	1
	8	3	52		17	24	1		2	0	0
	4	2	18		8	52	2		13	2	1
	8	1	58		18	40	0		17	1	1

4. Add 14 tuns, 42 gallons—3 hhds. 28 gal.—10 tuns, 2 hhds. 47 gal.—16 tuns, 1 hhd. 18 gal.—28 tuns, 48 gal.
70 tuns, 0 hhds. 57 gal.

5. Add 6 gal. 1 qt. 1 pint—18 gal. 3 qts.—28 gal. 2 qts. 1 pt.—3 qts. 1 pt.

6. Bought 7 casks of wine; the first two contained together, 46 gal. 2 qts. 1 pint—the next two, 51 gal. 3 qts.—the next two, 48 gal. 1 qt. 1 pt.—and the last, 22 gal. 2 qts. 1 pt. How much wine altogether? 169 gal. 1 qt. 1 pt.

7. Add 4 gal. 2 qts. 1 pt. 3 gills—3 qts. 1 pt. 2 gills—14 gal. 3 qts. 2 gills—6 gal. 1 qt. 1 pt. 2 gills.
26 gall. 3 qts. 1 pt. 1 gill.

DRY MEASURE.

	Bu.	pk.	qt.		bu.	pk.	qt.	pt.		pk.	qt.	pt.
1.—	74	2	2	2.—	18	2	6	1	3.—	5	6	1
	16	3	6		6	3	2	0		12	7	1
	2	1	5		26	1	7	1		19	3	0
	29	0	7		1	3	5	0		24	5	1

4. Add 6 bushels, 3 pecks, 5 quarts—2 pecks, 1 quart—18 bus. 6 quarts—32 bus. 2 pecks, 4 quarts—50 bus. 1 peck, 3 quarts. 108 bus. 2 pks. 3 qts.

5. Add 68 bus. 2 pecks, 4 quarts—31 bus. 6 qts.—28 bus. 1 pk. 7 qts.—14 bus. 3 pks. 5 qts. 143 bus. 0 pk. 6 qt.

6. A brewer has 68 bus. 3 pecks of barley—184 bus. 6 quarts—71 bus. 2 pecks, 5 qts.—84 bus. 2 pecks, 6 quarts. How much in all? 409 bus. 1 pk. 1 qt.

TIME.

	Y.	m.		w.	d.	h.	min.		d.	h.	min.	sec.
1.—	6	5	2.—	2	6	10	42	3.—	6	7	14	54
	7	11		5	3	14	16		9	22	45	30
	2	8		1	4	20	30		15	19	36	42
	9	9		7	6	23	59		7	4	7	41

4. Add 6 years, 4 months—5 years, 5 months—7 years, 11 months—12 years, 5 months. 32 y. 1 m.

5. If William is 32 years, 3 months, 14 days—Henry 30 years, 7 months, 22 days—Christopher 27 years, 26 days—Jervas 25 years, 10 months, 16 days; what is the sum of their ages? 115 y. 10 mo. 18 days.

6. What day of the year was the 5th of September, (9th month) 1785? the 248th.

7. Mercury revolves round the sun in 87 days, 23 hours, 15 minutes, 37 seconds—Venus in 224 days, 16 hours, 49 minutes, 12 seconds—the earth in 1 year, or 365 days—Mars in 1 year, 321 days, 23 hours, 30 minutes—Jupiter in 11 years, 314 days, 12 hours—Saturn in 29 years, 167 days, 5 hours. What is the sum of all those revolutions?

45 y. 21 d. 8 h. 34 min. 49 sec.

MOTION, OR CIRCLE MEASURE.

	Si.	°	′		°	′	″		Si.	°	′	″
1.—	3	18	52	2.—	17	42	30	3.—	2	14	25	37
	4	27	46		28	6	10		8	25	50	6
	7	20	30		49	52	58		9	29	59	1
	8	2	15		6	2	6		10	7	12	30

4. Add 2 signs, 13°—1 sign, 4° 15′ 58″—4 signs, 25° 22″—21° 19′ 39″. 9 si. 3° 35′ 59″.

5. Suppose the sun in 1 month goes 29° 50′ 32″—in another, 1 sign, 32′—in another, 1 sign, 1° 45′ 50″—in a fourth, 1 sign, 27′ 33″. How much is it altogether?

4 si. 2° 35′ 55″.

Application.

1. A man, at auction, buys of cloth, 17*l.* 3*s.* worth—of flannels, 20*l.* 15*s.* 7½*d.*—of linen, 41*l.* 6*s.* 4*d.*—of silks, 18*l.* 18*s.* 7¼*d.* What is the whole amount? 98*l.* 3*s.* 6¾*d.*

2. I am offered 12*l.* 16*s.* 9*d.* for my horse—6*l.* 2*s.* for my saddle and bridle—3*l.* 17*s.* 4½*d.* for some harness—and 62*l.* 18*s.* 10¼*d.* for my gig. If I sell all those articles, how much must I receive? 85*l.* 14*s.* 11¾*d.*

3. If a silver plate weigh 2 lb. 9 oz. 14 dwts.—another, 3 lb. 10 dwts.—another, 2 lb. 10 oz. 18 dwts.—and two more together, 5 lb. 3 oz. 12 dwts. What is the weight of the whole?

14 lb. 0 oz. 14 dwts.

4. Bought 4 feather beds: one weighed 1 qr. 22½ lb.—another, 1 qr. 16 lb.—another, 2 qrs. 10¾ lb.—and the fourth, 1 qr. 26¼ lb. What was the weight of the whole? 7 qrs. 19½ lb.

5. A man buys 5 loads of hay; the first weighs 1 ton, 3 cwt. 17 lb.—the second, 1 ton, 6 cwt. 1 qr.—the third and fourth together, 3 tons—the fifth, 19 cwt. 2 qrs. 17 lb. What is the weight of all? 6 tons, 9 cwt. 0 qr. 6 lb.

6. Lost at sea 4 hogsheads of sugar, weighing: the first, 6 cwt. 3 qrs. 24 lb.—the second, 10 cwt. 2 qrs. 18 lb.—the third, 14 cwt. 20 lb.—the fourth, 12 cwt. 2 qrs. What did they all weigh? 44 cwt. 1 qr. 6 lb.

7. At the same time were lost, 5 casks of gin: containing, No. 1, 84 gal. 2 qts.—No. 2, 63 gal. 2 qts. 1 pt.—No. 3, 71 gal. 3 qts.—No. 4, 73 gal. 1 qt. 1 pt.—and No. 5, 78 gal. 3 qts. 1 pt. What was the whole loss? 372 gal. 0 qt. 1 pt.

8. A little boy wished to buy chesnuts; the man who was selling them said to him: I have 5 quarts 1 pint in this bag—in that bag there are 7 quarts 1 pint—in the other bag, there are 7 quarts 1 pint—and in that other bag, 3 quarts 1 pint. Now if you can tell me how many pecks I have, I will give you a pint of chesnuts. The boy answered 3 pecks: did he win the chesnuts or not?

9. A merchant buys 3 pieces of carpet; the first contains 84 yds. 3 qrs.—the second, 102 yds. 1 qr. 2 nails—the third, 98 yds. 2 qrs. 3 nails. How much did he buy altogether, and what is the amount at 6¼ cents per nail?

Ans. 285 yds. 3 qrs. 1 na. Amt. $285.81¼.

10. A druggist bought a lot of salts, of which he sold 3 pounds, 10 ounces, 6 drams—then 4 pounds, 7 drams, 2 scruples—then 9 ounces, 5 drams, 1 scruple, 16 grains: he has yet 7 pounds, 2 ounces, 4 drams, 2 scruples, 4 grains. What did it weigh at first? An. 16 lb.

11. A grocer buys 4 bags of pepper: No. 1, weighs 1 qr. 22 lb. 10 oz. 6 dr.—No. 2, 2 qrs. 16 lb. 13 oz. 8 dr.—No. 3, 1 qr. 26 lb. 9 oz. 14 dr.—and No. 4, 1 qr. 18 lb. 7 oz. 15 dr. What is the weight of the whole? 8 qrs. 0 lb. 9 oz. 11 dr.

12. A man travels on foot 21 miles, 4 furlongs, in one day—the next day he walks 22 miles, 5 furlongs, 22 poles—the next day in a wagon he goes 31 miles, 6 furlongs, 32 poles—the next day on horseback 40½ miles. How far is he from where he started? 116 m. 4 fur. 14 p.

13. A farm consists of 6 inclosures; the first contains 42 acres, 3 roods, 36 poles—the second, 36 acres, 2 roods 24 poles—the third, 45 acres, 1 rood, 38 perches—the fourth, 52 acres, 2 roods, 28 perches—and each of the other two, contains 26 acres, 3 roods, 14 poles. How much land in all?

Ans. 231 a. 1 r. 34 p.

14. A man says to his son, a boy of 9 years 3 months old, I was born on the 5th of September (9th month) 1785; when I was 5 years, 3 months, and 25 days old I was sent to school, where I remained 11 years, 4 months. I then began to study my profession, and studied 6 years, 4 months, and 5 days; after exercising my profession during 5 years, 5 months, and 18 days, you were born; how old am I now? 37y. 8m. 18d.

15. In a field there are 4 piles of wood; the first contains 14 cords, 78 feet, 456 inches; the second, 27 cords, 115 feet, 1600 inches; the third, 16 cords, 124 feet, 1448 inches; and the fourth, 18 cords, 74 feet, 864 inches. How much in all? 78 cords 9 ft. 912 in.

16. I have 4 boxes: No. 1 contains 9 solid feet, 1454 inches; No. 2—1 cubic yard, 12 feet, 742 inches: No. 3—1 yard 21 feet, 1700 inches; No. 4—1 yard, 9 feet, and 1288 inches. They want to charge me freight for 5 cubic yards. Is that right?

17. If a star in 3 months, goes 2 signs, 24° 15′—in 4 other months, 3 signs 25° 52′ 45″—in 2 other months, 1 sign, 18° 25′ 15″—and in 3 other months, 3 signs, 21° 27′—has it accomplished a whole revolution?

COMPOUND SUBTRACTION.

249. Place the smaller number under the larger, so that the same denominations shall be under each other; then, beginning on the right, subtract the lower number from the one above and set down the remainder; pass them to the next denomination, and so on, till you have individually subtracted every denomination from the one above it. But if you cannot take the lower quantity from the one above, borrow one from the next higher denomination, reduce it into the denomination you are subtracting from; to this, add those already given; subtract the under number from that sum, set down the remainder, and when you pass to the denomination from which you borrowed, count the number as 1 less.—If, when you are forced to borrow, there is 0 at the next higher denomination, pass to the one above; and if 0 is there also, pass on to the next; and so on, till you can find 1 to borrow: convert that 1 into the next lower denomination; take 1 out of these, to convert into the next one; and so on, till you come to the denomination you are subtracting from, and then act as already directed; and when you come to the one you borrowed from, count it as 1 less.

M 2

250. N. B.—Every intervening 0 is to be considered as equal to as many of its own denomination, less one, as will make one unit of the next higher order. Examples will render this plain.

	l.	s.	d.
1st.	48	12	8
	29	12	4
	19	0	4

After placing pounds, shillings and pence under pounds, shillings and pence, beginning on the right, I say, 4*d.* from 8*d.* the remainder is 4*d*; then 12*s.* from 12*s.* 0 remains; 29*l.* from 48*l.* subtracted as in simple subtraction leave 19*l.* for remainder.

	l.	s.	d.
2d.	48	7	5
	9	6	$9\frac{1}{4}$

In this example I have $\frac{1}{4}$ in the subtrahend, and as I have none above, I must borrow 1*d.* which is equal to $\frac{4}{4}$; I then take $\frac{1}{4}$ from $\frac{4}{4}$ and find $\frac{3}{4}$ for remainder. Having borrowed 1*d.* from the 5, I have 4 left. 9*d.* cannot be taken from 4*d.* I must then borrow; but what shall I borrow? 1 shilling, which is equal to 12*d.*; to these 12*d.* adding the 4 I have already, and from the sum 16, taking off 9*d.* I find 7*d.* is left. From 7*s.* having borrowed 1, I must count the 7 only as 6; and having 6*s.* to subtract from it 0 remains; and 9*l.* from 48*l.* leaves 39*l.*

	l.	s.	d.
3d.	30	0	$7\frac{1}{4}$
	9	18	$8\frac{3}{4}$

In this example, $\frac{3}{4}$ in the subtrahend, cannot be taken from the $\frac{1}{4}$ above; I then borrow 1*d.* equal to $\frac{4}{4}$, this added to $\frac{1}{4}=\frac{5}{4}$, from this taking off $\frac{3}{4}$, $\frac{2}{4}$ remain. I cannot take 8*d.* from 6*d.* I have then to borrow from the shillings; but as there are none, I pass on to the pounds, and as I find no unit of pounds, I borrow 1 from the 3 tens, and leaving 9 units with the 0, I convert the other one into 20 shillings; out of these 20 shillings I borrow 1, convert it into 12 pence, and adding the 6 I already have, I obtain 18 pence, from which taking 8, the remainder is 10. Now from 19 shillings taking 18, 1 remains, and taking 9 pounds from 29, the remainder is 20 pounds.

	l.	s.	d.
4th.	10	0	0
	6	14	6*d.*$\frac{2}{4}$

In this case, (No. 4) having no farthings, no pence, no shillings, I borrow 1 from the pounds, and convert it into 20 shillings; leaving 19 of these, I convert the other into 12 pence, and 1 of these into 4 farthings, out of which taking 2, there remain 2—6 pence from 11, leave 5; 14 shillings from 19, leave 5; and 6 pounds from 9, leave 3.

	l.	s.	d.		l.	s.	d.		l.	s.	d.		l.	s.	d.
5.—From	5	3	8	6.	15	7	5	7.	18	0	0	8.	23	0	$8\frac{3}{4}$
											$10\frac{3}{4}$		14	16	$5\frac{1}{2}$.

1. Height of At.? uniform density? Pressure? Amount of? 15 lb 14 tons— Proof of? Barometer — Column Mercury — Water? why are we not crushed? Thermometer — Is it on Same principle? How is Sound occasioned? Rate —

Light. Velocity — Reflection, what Surfaces reflect best? why do we see ourselves in a Glass? Refraction? — Eye — near Sighted? far Sighted, Colors — Red, Yellow, blue — How do we See bodies — why a Rose red. Violet — blue?

l 6 pence, from 24

½ pence, from 200

75l. 16s. $10\frac{3}{4}$d.

ls, 17 shillings, $11\frac{1}{4}$

75l. 2s. $0\frac{3}{4}$d.

shillings, $6\frac{3}{4}$ pence,

23l. 7s. $5\frac{1}{4}$d.

	lb.	oz.	dwt.	gr.
—	25	0	0	0
	7	3	17	20

z. 10 dwt.

6 lb. 2 oz. 10 dwt.

8 lb. 10 oz. 15 dwt.

. 4 oz. 17 dwt. 2 gr.

metal, and find af-
uch was lost in the
11 oz. 8 dwt. 22 gr.
ing 6 oz. 14 dwt.;
, and working them
veight to be 1 lb. 2
en added?

1 oz. 4 dwt. 8 gr.

741
378

741

									qr.	lb.	oz.	dr.
1.—From	21	12	1	2.—	17	2	14	3.—	3	21	14	7
Take	6	18	3		4	1	19			23	14	12

4. From 38 cwt. 3 qrs. 10 lb. (weight of a wagon load of hay,) subtract 17 cwt. 3 qrs. 22 lb. for the weight of the waggon; how much hay is there? 20 cwt. 3 qrs. 16 lb.
5. From 7 cwt. 1 qr. 20 lb. take 2 cwt. 26 lb.

5 cwt. 0 qr. 22 lb.

6. From 2 qrs. 13 lb. 10 oz. take 1 qr. 11 oz. 12 dr.
1 qr. 12 lb. 14 oz. 4 dwt.

7. A merchant who had 78 boxes of sugar, weighing together 246 cwt. 2 qrs. 18 lb.—sold 49 boxes weighing 157 cwt. 3 qrs. 25 lb.—How many boxes had he yet on hand; and what was their weight? 29 boxes, weighing 88 cwt. 2 qr. 21 lb.

	℥	ʒ	℈
1.—From	11	5	2
Take	6	6	2

	℥	ʒ	℈	*gr.*
2.—	8	5	0	14
	2	5	2	18

	ʒ	℈	*gr.*
3.—	7	0	0
	5	1	6

4. From 19 ℔ 8 ℥ 4 ʒ take 16 ℔ 11 ℥ 7 ʒ. 2 ℔ 8 ℥ 5 ʒ.

5. From 35 ℔ 7 ℥ 3 ʒ 1 ℈ 14 *gr.* take 14 ℔ 10 ℥ 0 ʒ 1 ℈ 19 *gr.* 20 ℔ 9 ℥ 2 ʒ 2 ℈ 15 *gr.*

6. From 7 ℥ take 2 ℥ 6 ʒ and 14 *gr.* 4 ℥ 5 ʒ 2 ℈ 6 *gr.*

7. Bought 4 ʒ of calomel; after giving 2 doses of 14 *gr.* each, and 1 dose of 16 *gr.*; how much have I left?
3 ʒ 0 ℈ 16 *gr.*

	L.	*M.*	*F.*	*P.*
1.—From	20	2	6	31
Take	14	2	7	16

	yds.	*ft.*	*in.*	*b.c.*
2.	16	0	2	2
	4	2	10	1

	deg.	*g.m.*
3.	7	12
	4	48

4. From 7 miles, 2 furlongs, 14 poles, take 4 miles, 4 furlongs, 4 poles. 2 m. 6 f. 10 p.

5. From 2 leagues, take 5 miles, 6 fur. 7 poles. 1 f. 33 p.

6. There is a piece of turnpike of 17 yards, 1 foot, 4 inches to be made. John agrees to make $8\frac{1}{2}$ yards of it; how much will remain undone? 8 yds. 2 ft. 10 in.

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	yds.	*qrs.*	*na.*
1.—From	15	2	0
Take	6	3	2

	E.E.	*qrs.*	*na.*
2.—	22	4	1
	14	4	3

	E.Fr.	*qrs.*	*na.*
3.—	19	2	2
	5	5	2

4. From 16 yards, take 5 yds. 2 qrs. 2 nails. 10 y. 1 q. 2 n.

5. A merchant sells 6 yds. 2 qrs. of flannel to A; 7 yds. 3 qrs. 3 nails to B, out of a piece which contained 22 yds. 2 *qrs.*; how much remains to him? 14 y. 2 qrs. 1 n.

6. *From* $24\frac{1}{2}$ English ells, subtract 15 yds. 3 qrs.
59 qrs. 2 nails.

	A.	*R.*	*P.*		*M.*	*A.*	*R.*		*yds.*	*ft.*	*in.*
1.—From	14	2	26	2.—	1	342	2	3.—	14	3	42
Take	6	2	31		0	500	0		10	8	100

4. From 124 acres, 3 roods, take 96 acres, 3 roods, 24 perches. 27 A. 3 R. 16 per.

5. From 1 square mile, take 284 acres, 3 roods and 14 perches. 355 A. 0 R. 26 per.

6. If, on a lot containing 888 square yards, 8 feet, I build on 109 yards, 8 feet, 89 inches; how much ground remains? 778 yds. 8 ft. 55 in.

	Cord.	*feet.*		*yds.*	*ft.*		*yds.*	*ft.*	*in.*
1.—From	12		2.—	12	14	3.—	40	24	1400
Take	7	100		9	22		32	18	782

4. From 3 loads of round timber, take 1 load and 17 feet. 1 load, 23 feet.

5. A farmer delivers me $4\frac{1}{2}$ cords of wood, he was to have furnished me 7 cords and 82 feet. How much has he to bring yet? 2 cords, 18 ft.

6. If, from a pile of stones, containing 27 yds. 14 feet, 340 inches, you take 19 yds. 24 feet, 864 inches; how much will remain? 7 yds. 16 ft. 1204 in.

	Tun	*hhd.*	*gal.*		*hhd.*	*gal.*	*qt.*		*hhd.*	*gal.*	*qt.*	*pt.*
1.—From	18	2	40	2.—	32	41	0	3.—	9	18	0	0
Take	14	3	52		7	51	2		6	27	2	1

4. From 1 hhd. take 52 gal. 2 qts. 1 pt. 10 g. 1 qt. 1 pt.

5. From 1 tun of wine, take 2 hhd. 20 gal. 3 qts.; what remains? 1 hhd. 42 gal. 1 qt.

6. A storekeeper puts 58 gal. cordial into a cask; the last time he draws, he finds only 3 gal. 2 qts. 1 pt. 2 gills; how much had he sold before? 54 gal. 1 qt. 0 pt. 2 gills.

	Bu.	*pk.*	*qt.*		*bu.*	*pk.*	*qt.*		*pk.*	*qt.*	*pt.*
1.—From	14	2	1	2.—	27	0	0	3.—	5	5	0
Take	8	3	6		12	2	4		4	7	1

4. From 200 bus. 6 qts. take 87 bus. 2 pecks. 112 bus. 2 pk. 6 qts.

5. Subtract 18 bus. 3 pecks, 5 qts. from 21 bus. 1 peck.
2 bus. 1 pk, 3 qts.

6. A farmer brought 18 bus. 2 pecks of potatoes; after selling 9 bus. 3 pecks, and 6 qts., how many had he left?

8 bus. 2 pk. 2 qt.

7. A boy picked up 5 pecks 3 qts. of chesnuts; he gave 2 qts. and 1 pt. to his sister, and sold 3 pecks, 4 qts. to a neighbour; how many had he left? 1 pk. 4 qts. 1 pt.

	W.	*d.*	*h.*		*d.*	*h.*	*m.*		*d.*	*h.*	*m.*	*sec.*
1.—From	17	4	18	2.—	27	8	53	3.—	54	12	40	15
Take	14	5	21		21	12	50		37	6	52	46

251. NOTE.—When years, months, and days are connected together; to find the difference between two given dates; after placing the less under the greater, if the lower number of days be greater than the upper, borrow 1 from the months, (lower number,) count it for as many days as there are in the borrowed month; take the lower number from it, add the difference to the upper number, and set down the amount; count the upper months as 1 less, and subtract as in the following example.

4. A man says, "my oldest son was born on the 17th of August, (or 8th month) 1804; my youngest, on the 12th of March, 1817,(3d month). What is the difference of their ages?"

	y.	*m.*	*d.*
greater,	1817	3	12
less,	1804	8	17
	12	6	26

As I cannot take 17 days from 12, I borrow 1 from the 8, and as that month is the month of August, (or 8th month) which has 31 days, I take 17 from 31, and to the difference 14, adding 12, I set down the sum 26, and counting the upper months as 1 less, I borrow 1 year, equal to 12 months, and 2 make 14; taking 8 from it, I set down 6, and find 12 for the difference of years.

5. A person was born on the 25th of January, (1st month) 1801; how old was he on the 22d of November, (11th month) 1818? 17 y. 9 m. 28 d.

6. A man was born on the 15th of February, 1796; how old is he on the 14th October, (10th month) 1820?

23 y. 7 m. 28 d.

7. A bond was given on the 27th May, (5th month) 1814; *it is to* be paid on the 9th Sept. (9th month) 1822; for what *time must* interest be calculated thereon? 8 y. 3 m. 13 d.

	°	′	″		si.	°	′		cir.	si.	°
1. From	142	14	50	2.	7	15	20	3.	1	0	0
Take	78	46	27		4	27	43		0	7	22

4. A city is 47° 12′ 41″ east longitude; another is 39° 37′ 15″; what is their difference of longitude? 7° 35′ 26″

5. Philadeldhia being 75° 8′ W. longitude from London, and Boston 70° 52″, what is their difference of longitude? 4° 16′.

6. Pennsylvania being situated between 39° 43′ and 42° N. latitude; how many degrees is the southern from the northern boundaries? 2° 17′.

7. When a planet has gone through 8 signs 22° 42′ 13″, how far is it short of a complete revolution? 3 si. 7° 18′ 47″.

COMPOUND ADDITION AND SUBTRACTION.

PROMISCUOUS EXAMPLES.

1. A man owed me 14*l.* 14*s.*, he paid me 3*l.* 6*s.* 4*d.*, I then bought of him for 5*l.* 12*s.* 6*d.*; how much does he owe me yet? 5*l.* 15*s.* 2*d.*

2. The distance from New York to Philadelphia was formerly as follows: from New York to Newark, 8 m. 7 fur. 28 per.—from thence to Elizabethtown, 6 m. 3 fur.—from thence to Brunswick, 19 m. 6 fur. 31 per.—from thence to Princeton, 18 m. 4 fur. 33 per.—from Princeton to Trenton, 12 m. 9 per.—from Trenton to Bristol, 10 m. 2 fur. 25 per. and from Bristol to Philadelphia, 20 m. 1 fur. 14 perches. Now, it is reduced, from Philadelphia to Powleshook, or Jersey city, to 85 m. 3 fur. 22 perches, exclusive of the breadth of the river, which is found by measurement to be 1 m. 6 per.: how much shorter is the road? 9 m. 6 fur. 32 per.

3. A man buys a lot of ground which he supposes to contain 48 sq. yds. 4 ft. 80 in. after measuring it, he finds only 45 yds. 6 ft. 112 in. what is the deficiency? 2 sq. yds. 6 ft. 12 in.

4. Bought 4 bags of hops; Nos. 1 and 2, each weigh 3 qrs. 24 lb.; No. 3, weighs 3 qrs. 17 lb.; and No. 4, 3 qrs.—Among the whole, I find 1 qr. 21 lb. damaged; having sold 2 cwt. 2 qrs. 18 lb. how much have I on hand? 1 qr. 26 lb.

5. Philadelphia is 75° 8′ W. long. from London; St. Petersburg (Russia) is 30° 19′ E.; what is the difference of longi-

tude between those two cities; or in other words, how many degrees longitude are there between them? 105° 27′.

6. The longitude from Paris to Constantinople being 26° 39′—and from London to Constantinople 28° 58′; how can I find the difference of longitude between Paris and London; and what is it? 2° 19′.

7. A, contracts for 15 cords of wood. He at one time receives 3 cords, 48 feet; at another, 4 cords, 106 feet; at another 2 cords, 100 feet; how much must he yet receive?

4 c. 2 ft.

8. A farmer has 120 bus. 3 pecks of wheat in a granary; 85 bus. 1 pk. in another; he sells 57 bus 2 pecks, and sends 15 bus. 3 pecks to the mill; how much has he yet?

132 bus. 3 pks.

9. If I mix two drugs; 4 ℔ 7 ℥ 6 ʒ 1 ℈ 14 gr. of each, and from the composition sell 7 ℔ 8 ℥ 6 ʒ 1 ℈ 15 gr.; what remains? 1 ℔ 6 ℥ 6 ʒ 1 ℈ 13 gr.

10. If, from a load of round timber, 7 feet, 1248 inches are found rotten; what will be the remainder? 32 ft. 480 in.

11. A manufacturer finishes 3 pieces of grey cloth; two of them contain each 31 yds. 3 qrs.—and the other 35 yds. 3 qrs. He sells in the first place, 15 yds; in the second place, 42 yds. 0 qrs. 2 nails. He says he has yet 42 yds. and 2 nails. Is he right?

12. The revolutionary war in America, began April 19th, (4th month) 1775, it ended January 20th, (1st month) 1783; how long did it contiuue? 7 y. 9 mo. 1 d.

13. The planet Venus revolves round the sun in 224 days, 16 hours, 49 minutes, 24 seconds; and Mercury, in 87 days, 23 hours, 15 minutes, 53 seconds; what is the difference of their periodical revolutions? 136 d. 17 h. 33 min. 31 sec.

14. A merchant deposits in bank, two thousand, six hundred dollars; afterwards three hundred and forty-seven dollars, forty three cents; he then draws out one thousand, four hundred and eighty-nine dollars, fifty-nine cents; he then puts in seven hundred and ninety-four dollars, fifteen cents; draws out again, two thousand, eight dollars; after which, he says he has in bank, a balance of two hundred and forty-three dollars, ninety-nine cents. Is he correct?

15. I give old silver to a silver-smith, weighing 14 oz. 15 dwt. 9 gr. he returns articles weighing 1 lb. 9 oz. 18 dwt. 12 gr.; what is the difference of the two quantities?

7 oz. 3 dwt. 3 gr.

COMPOUND MULTIPLICATION.

252. COMPOUND MULTIPLICATION teaches to multiply numbers consisting of several denominations, by numbers consisting only of one.

When the multiplier does not exceed 12, work by

RULE 1.

253. Place the multiplier under the lowest denomination of the multiplicand, then multiply successively each denomination of the multiplicand, by the multiplier; reduce each individual product to the next higher denomination; carry the quotient to that denomination, and set down the remainder under its respective column.

PROOF.

254. Double the multiplicand, and multiply it by half the multiplier.

EXAMPLES.

MONEY.

	l.	*s.*	*d.*		*l.*	*s.*	*d.*		*l.*	*s.*	*d.*
1.—	4	8	7¼	2.—	14	16	9½	3.—	12	16	9¾
			8				7				4
	35	8	10		103	17	6½		51	7	3

				Ans.
4.	5 pounds of tea, at 7*s.* 6*d.*	1*l.*	17*s.*	6*d.*
5.	7 gallons of brandy, 14*s.* 8*d.*	5	2	8
6.	6 barrels of beer, 1*l.* 12*s.* 7½*d.*	9	15	9
7.	10 hundred weight of iron, 1*l.* 9*s.* 9¾*d.*	14	18	1¾
8.	9 cheeses, 1*l.* 13*s.* 5*d.*	15	0	9
9.	12 quintals of fish, 18*s.* 4¼*d.*	11	0	3
10.	11 pounds of carmine, 2*l.* 11*s.* 11*d.*	28	11	1
11.	8 bushels of wheat, 6*s.* 8*d.*	2	13	4

WEIGHTS AND MEASURES.

	T.	*cwt.*	*qr.*	*lb.*	*dr.*		*lb.*	*oz.*	*dwt.*	*gr.*		℔	℥	ʒ	℈	*gr.*
1.—	2	15	1	22	15	2.—	5	11	17	8	3.—	2	9	7	2	14
					6					3						8

	L.	*M.*	*fur.*	*P.*		*yd.*	*ft.*	*in.*		*yd.*	*qr.*	*na.*		*EE.*	*qr.*	*na.*
4.—	8	2	6	26	5.—	14	1	9	6.—	18	3	2	7.—	28	4	3
				5				9				12				7

	M.	*A.*			
8.—	3	520			
		11			

	A.	*R.*	*P.*	
9.—	14	3	38	
			10	

	T.	*hhd.*	*gal.*	*qt.*	*pt.*
10.—	7	3	51	3	1
					5

	Gal.	*qt.*	*pt.*	*gills.*
11.—	8	2	0	3
				4

	sq.yd.	*ft.*	*in.*
12.—	4	7	112
			6

	cub.yd.	*ft.*	*in.*
13.—	17	24	1414
			12

	Bu.	*pk.*	*qt.*
14.—	7	2	6
			9

	w.	*d.*	*h.*	*min.*	*sec.*
15.—	3	6	19	43	54
					7

To Multiply by Fractional parts, as $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, *&c.*

255. RULE.—Multiply the *price* by the upper figure of the fraction, and divide the *product* by the lower, for the answer. This is evident, from what we have already said on the multiplication of fractions, (art. 164,) and although *compound* division is to be performed, it is so simple, that we do not hesitate to offer this rule previously to the introduction of division.

EXAMPLES.

1. What will $\frac{3}{4}$ of a yard of cloth be worth, at 2*l.* 12*s.* 8*d.* per yard?

	l.	*s.*	*d.*	
	2	12	8	
			3	upper number or numerator.
Denominator 4)	7	18	0	
	1	19	6	

To divide the product 7*l.* 18*s.* by 4, I say: 4 into 7 pounds goes once, and 3 pounds over; I set down 1, and reducing the 3 remaining pounds into shillings, and adding the 18 shillings I already have, I obtain 78 shillings, which being divided by 4, give 19 shillings for quotient, and 2 shillings for remainder; these being reduced to pence, are equal to 24 pence, dividing them by 4, I find 6 pence.

256. NOTE.—When the numerator is 1, merely divide the *given price* by the denominator.

2. What is $\frac{1}{4}$ of a yard of silk velvet at 1*l.* 10*s.* per yard?

	l.	*s.*	*d.*
4)	1	10	0
	0	7	6

3. $\frac{3}{8}$ of a yard of cambric, at 17*s.* per yard? 0*l.* 6*s.* 4$\frac{1}{2}$*d.*
4. $\frac{3}{4}$ of a yard of cloth, at 3*l.* 6*s.* per yard? 1 18 0
5. $\frac{5}{8}$ of a yard of merino trimming, at 3*s.* 4*d.*? 0 2 1
6. $\frac{3}{4}$ of a yard of mulmull, at 2*l.* 7*s.* 4*d.* per yd.? 1 15 6
7. $\frac{7}{8}$ of a yard of fine linen, at 8*s.* 6*d.* per yd.? 0 7 5$\frac{1}{4}$
8. $\frac{1}{8}$ of a yard of silk velvet, at 1*l.* 13*s.* 2*d.*? 0 4 1$\frac{3}{4}$
9. $\frac{3}{8}$ of a yard of fine lace, at $2.50? $0 93$\frac{3}{4}$

When the multiplier exceeds 12, and is the product of two factors or numbers in the multiplication table, work by

RULE 2.

257. Multiply the given price by one of said factors, and the product by the other.

PROOF.—Change the factors.

EXAMPLES.

	l.	*s.*	*d.*			*l.*	*s.*	*d.*
1.—18 yd. at	1	12	5$\frac{1}{2}$?	18=6×3	Proof	1	12	5$\frac{1}{2}$
			6					3
	9	14	9			4	17	4$\frac{1}{2}$
			3					6
	29	4	3			29	4	3

2. 16 yards silk, at 4*s.* 9*d.* 3*l.* 16*s.* 0*d.*
3. 14 casks wine, at 3*l.* 2*s.* 6$\frac{1}{2}$*d.* 43 15 7
4. 45 pair shoes, 7*s.* 6*d.* 16 17 6
5. 100 gallons brandy, 1*l.* 9*s.* 8$\frac{3}{4}$*d.* 148 12 11
6. 63 yards cambric, 9*s.* 3*d.* 29 2 9
7. 56 barrels of flour, each containing 1 cwt. 3 qr. 98 cwt.
8. 84 pieces of galoon, each containing 31 yards, 2 quarters, 2 nails. 2656 yd. 2 qr.
9. 3 dozen fowls, at 3*s.* 9*d.* a pair. 6*l.* 15*s.*
10. 120 casks of wine, each containing 42 gallons, 3 quarts, 1 pint. 5145 gal.

When the multiplier is not the exact product of two tabular numbers, use

RULE 3.

258. Multiply by the two numbers which come nearest to it, and by the deficiency or excess, multiply the multiplicand. This third product added to or subtracted from the second one, will give the answer.

EXAMPLES.

1. 47 yards baize, at 4*s.* 7*d.* per yard.

$47=9\times5+2$				$47=8\times6-1$		
l.	*s.*	*d.*		*l.*	*s.*	*d.*
0	4	7×2		0	4	7
		9				8
2	1	3		1	16	8
		5				6
10	6	3		11	0	0
+	9	2		—	4	7
10	15	5		10	15	5

The one may serve to prove the other, or else, change the factors.

2. 37 bushels of corn, at 4*s.* 11*d.* Ans. 9*l.* 1*s.* 11*d.*
3. 51 pounds of tea, at 3*s.* 6*d.* 8 18 6
4. 23½ barrels of fish, 2*l.* 4*s.* 7*d.* 52 7 8½
5. 57½ gallons of rum, 4*s.* 2½*d.* 12 1 11¾
6. 130 yards of cloth, 2*l.* 3*s.* 9*d.* 284 7 6
7. 31 pieces of cambric, 3*l.* 13*s.* 4*d.* 113 13 4
8. 125¾ yards silk, 5*s.* 7*d.* 35 2 1¼
9. 17 doses of medicine, each containing 3 ʒ 1 ℈ 14 *gr.*
7 ℥ 4 ʒ 1 ℈ 18 *gr.*
10. 119 demi-johns, each containing 1 gallon, 3 quarts, 1 pint. 223 gal. 0 qr. 1 pt.

When the multiplier is greater than the product of any two tabular numbers, use

RULE 4.

259. Multiply by as many tens, *less one*, as there are figures in the multiplier; then multiply the last product by the left *hand figure* of the multiplier; again, multiply the multipli-

cand by the units of the multiplier; afterwards the pro of the first ten, by the tens of the multiplier, &c. plac several products as in addition, and sum them up fo answer.

EXAMPLES.

1. What will 364 yards of lace cost, at 1*l.* 7*s.* 8*d.*

	l.	*s.*	*d.*	
	1	7	8	×4, units of multiplier.
			10	
Product of 1st ten	13	16	8	×6, tens of multiplier.
			10	
	138	6	8	price of 100 yards.
			3	hundreds of multiplier.
	415	0	0	price of 300 yards.
	5	10	8	price of 4 yards.
	83	0	0	price of 60 yards.
Ans.	503	10	8	price of 364 yards.

2. 2375½ gallons of molasses, at 3*s.* 5¼*d.*

	l.	*s.*	*d.*	
	0	3	5¼	×5 and by ½
			10	
	1	14	7	×7
			10	
	17	5	10	×3
			10	
	172	18	4	
			2	
	345	16	8	price of 2000 gallons.
	0	17	3½	5
	12	2	1	70
	51	17	6	300
		1	8¾	0½
Ans.	410	15	3¼	

3. 178 yards muslin, at 4*s.* 5*d.* per yard. Ans. 39*l.* 6*s.* 2*d.*
4. 276 pounds of coffee, at 1*s.* 7½*d.* 22 8 6
5. 563 yards of cloth, 1*l.* 6*s.* 7*d.* 748 6 5
6. 284½ yards muslin, 3*s.* 9½*d.* 53 18 8¾
7. 460½ boards, each 3 yards, 2 feet, 8 inches.
1790 yd. 2 ft. 6 in.
8. 177 chests of goods, each 2 hundred weight, 3 quarters, 14 pounds. 508 cwt. 3 qr. 14 lb.
9. 52,308 feet of boards, at 3¼*d.* per foot. 708*l.* 6*s.* 9*d.*
10. 89 pieces of lace, each 18 yards, 3 quarters, 1 nail.
1,674 yd. 1 qr. 1 na.
11. 17 loaves, each 4 lb. 5 oz. 8¼ dr. 75 lb. 8 oz. 0¼dr.

Application.

1. What is the price of 5 yards dimity, at 5*s.* 4¼*d.*?
1*l.* 6*s.* 10¼*d.*
2. What is the price of 7 yards muslin, at 9*s.* 4½*d.*?
3*l.* 5*s.* 7½*d.*
3. What will 9 barrels of flour cost, at 1*l.* 11*s.* 5*d.*?
14*l.* 2*s.* 9*d.*
4. What is the weight of 548 barrels of flour, each 1 cwt. 3 quarters? 959 cwt.
5. What is the value of 94 dollars, at 7*s.* 6*d.* (Pennsylvania currency)? 35*l.* 5*s.*
6. What is the value of 94 dollars, at 6*s.* (New England currency)? 28*l.* 4*s.*
7. What is the weight of 487 dollars, at 17 pennyweights, 6 grains each? 35 lb. 0 oz. 0 dwt. 18 gr.
8. A gentleman, whose income is 250*l.* spends daily 9*s.* 6½*d.* How much does he spend in one year, or 365 days? and how much does he save yearly?
He spends 174*l.* 2*s.* 8½*d.* and saves 75*l.* 17*s.* 3½*d.*
9. What is the weight of 216 bales of cotton, each 3 hundred, 2 quarters and 15 pounds after deducting 8 pounds, 2 ounces, for the weight of each wrapper?
10. How many yards in 75 perches, each being 30 yards, 1 quarter long?
11. In 139 degrees how many miles, each containing 69 miles, 4 furlongs?
12. Bought 39 pieces of ribbon, each containing 15 French ells, 4½ quarters; how many yards altogether, and what is their value, at 5¼*d.* per yard?

13. I. Manners

Bought of P. Vender,

$23\frac{3}{4}$ pounds of coffee,	at 1*s*. $10\frac{1}{2}$*d*.		2*l*. 4*s*. $6\frac{1}{4}$*d*.
$57\frac{1}{2}$ gallons of rum,	4 $2\frac{1}{2}$		12 1 $11\frac{3}{4}$
183 gallons of gin,	7 5		
121 bushels of corn,	4 3		
			107*l*. 18*s*. 0*d*.

14. R. Fairchild

Bought of H. Plyant,

$18\frac{3}{4}$ yards of linen,	at \$0.69$\frac{1}{2}$	
16 yards of silk,	1.12	
114 pair of gloves,	0.65	
78 pieces of ribbon,	2.15	
		\$272.82$\frac{1}{4}$

15. How many bushels of apples in 86 barrels, containing each 2 bushels, 3 pecks, 4 quarts?

16. One year being equal to 365 days, 5 hours, 48 minutes, 57 seconds; how many days will there be in a century or 100 years? 36,524 d. 5 h. 35 min.

COMPOUND DIVISION.

260. Compound division teaches to divide numbers consisting of several denominations.

We will divide it into two cases. 1st, When the divisor has only *one* denomination. 2d, When it has *several* denominations.

Case 1.

261. ***When the divisor has but one denomination, and when the quotient is like the dividend.***

GENERAL RULE.

1st, Divide the integers of the dividend by the divisor; reduce the remainder to the next lower denomination; add to the product those of the same denomination, found in the dividend; divide that sum by the same given divisor; convert the second remainder into the next lower denomination; add

those given; divide again by the same divisor; and so on, until you obtain the quotient of the last or lowest denomination. (This reduction is made according to the third case of simple reduction: art. 220.)

262. If any of the remainders, after being reduced to the next denomination, should not contain the divisor, put 0 in the quotient, reduce immediately to the next lower, and continue the operation.

PROOF.

263. Multiply (as in simple division) the quotient by the divisor, and the product will be the dividend.

EXAMPLE.

1. If 17 yards of cloth cost 23*l.* 0*s.* 4½*d.* what is it per yard?

```
17) 23l. 0s. 4½d.(1l. 7s. 0¾d.
    17
    ——
     6
    20
    ——
17)120(7s.
   119
   ———
     1
    12
    ——
17) 16(0d.
     4
    ——
17) 66(¾
    51
    ——
    15
```

I first divide the 23*l.* by 17; the quotient is 1, and the remainder 6. As the divisor is no longer contained in the integers of the dividend, I reduce the 6 remaining pounds into 120 shillings, and as there are none in the dividend, I divide these by the same divisor 17, and obtain for quotient 7 shillings. The remainder 1 shilling, being converted into pence, and increased by the 4*d.* found in the dividend, gives 16 pence. As these do not contain the divisor 17, I place 0 pence in the quotient. Now reducing these 16 pence into farthings, and adding the 2 given in the dividend, I find 66 quarters, which being divided by 17, produce 3 farthings, and a remainder which I entirely neglect.

264. NOTE.—It is evident, that by multiplying the successive remainders by 20, by 12, and 4, we make the dividends first 20, then 12, then 4 times greater; the quotient would, of course, be too great (art. 60,) should we consider it as integers, but by making it express subdivisions of the integers,

that is, quantities, first 20, then 12, then 4 times *less*, we counterbalance the increase, and obtain the true quotient.

2. Divide	61*l.*	14*s.*	6*d.*	by 6		Ans. 10*l.* 5*s.* 9*d.*
3. "	2	9	8	5		0 9 11¼
4. "	14	2	9	9		1 11 5
5. "	11	0	3	12		0 18 4¼
6. "	5	2	8	7		0 14 8

7. 38T. 3 hh. 7gal. 1qt. 1pt. by 5 — 7 T. 3hh. 51gal. 3 qt. 1pt.
8. 33 gal. 1qt. 1 pt. 0 gill. by 7 — 8 gal. 2 qt. 0 pt. 3 gills.
9. 5145 gal. by 120 — 42 gal. 3 qt. 1 pt.
10. 7℥ 4ʒ 1℈ 18 *gr.* by 17 — 3ʒ 1℈ 14 *gr.*
11. 1674 yd. 1 qr. 1 na. by 89 — 18 yd. 3 qr. 1 na.
12. 508 cwt. 3 qr. 14 lb. by 177 — 2 cwt. 3 qr. 14 lb.
13. 74 Fr. E. 4½ qr. by 26 — 2 Fr. E. 5¼ qr.
14. 2268 yd. 3 qr. by 75 — 30 yd. 1 qr.

265. NOTE.—It is deemed more expedient to divide at once by the divisor itself, than by its composite numbers; because the operation is not much shortened by it, whilst the working of the remainders is always perplexing and unintelligible.

RULE 2.

266. When the divisor has but one denomination, and when the quotient is not of the same kind as the dividend, attend to the following directions previous to performing the division.

267. Reduce the dividend to its lowest denomination, and afterwards multiply the divisor by the numbers which were used to reduce the dividend; or, by their product. This is done in order to increase the dividend and divisor equally, so as not to alter the quotient. Then you have an *incomplex* number to divide by another; that is, to perform common division, by which you obtain the integers of the quotient; to get the other denominations, reduce the remainder of the dividend to the next denomination the quotient is to express, and the successive remainders to the other denominations.

EXAMPLE.

1. How many yards can be bought for 63*l.* 16*s.* 4½*d.* at the rate of 3*l.* per yard?

It is evident, that as many times one yard can be bought, as the *price* of one yard is contained in the money to be expended. The quotient then, is to express yards, and parts

of yards; of course, it is not like the dividend which expresses pounds, &c.

```
3)   63l. 16s. 4¼d(
20   20
--   ----
60   1276
12     12
---  -----
720  15316
  4      4
---- -----
2880)61266(21 yd. 1 qr. 0 na.
      5760
      ----
      3666
      2880
      ----
       786
         4
      ----
2880)3144(1
     2880
     ----
      264
        4
     ----
2880)1056(0
```

I first reduce the dividend to the lowest denomination, that is, to 61266 farthings; I then multiply the divisor by the same numbers 20, 12, and 4, used to reduce the dividend, and have 61266 to divide by 2880. The quotient 21, must express yards, according to the conditions of the question. In order to have the subdivisions of the yard, I reduce 786, remainder of the dividend, into quarters, and dividing again by 2880, I get 1 quarter; I then reduce the remainder 264, into nails, and as 1056 does not contain 2880, there is 0 nail in the quotient.

2. If 1 ton of hay cost 8*l*.; how many tons can be bought for 131*l*. 7*s*. 8¼*d*.? 16 T. 8 cwt. 1 qr. 23 lb.

3. 1 peck of apples sells for 42 cents; how many pecks can I have for 7 dollars, 51 cents? 17 pk. 7 qt.

4. How many gallons of wine can I buy for 22 dollars, at the rate of 2 dollars, 75 cents per gallon? 8 gal.

5. For 17*l*. 2*s*. 8*d*. how many yards can I have, at the rate of 6*l*. per yard? 2 yd. 3 qr. 1 na. +

6. For 314 dollars, 50 cents; how many cords of wood can I buy, at 5 dollars, 15 cents per cord? 61 cords. +

7. How many pounds of cheese can I buy for 5 dollars, 28 cents, at 11 cents per pound? 48 lb.

8. How many hundred weight of hay can I have for 17 dollars, 82 cents, at 88 cents per hundred weight? 20cwt. 1qr.

9. How long can I hire a man for 75 dollars, at 100 dollars per year? 9 months.

10. How far can I send 17 tons of goods, at the rate of 11 shillings per mile, for 7*l.* 14*s.* 6¼*d.*? 14 miles. +

267. When in reducing a sum to its lowest denomination, several fractions occur, it frequently happens, that it cannot be proved by ascending reduction, as for instance; Ex. 9, of land measure (page 123.)—In such a case, perform compound division.

Take, for dividend, the given number of the lowest denomination, and for divisor, the integer reduced to the same denomination as the given number, and then divide as per 2d rule of compound division.

The 9th ex. alluded to, is: reduce 34 square perches, 8 yards, and 6 feet, to square inches.

The result is 1,344,168 square inches. If you try to prove it by ascending reduction, you do not obtain the primitive number 34 square perches, 8 yards, 6 feet—in these cases, use compound division, which is always the *surest proof.*

Bring 1,344,168 square inches to square perches.

```
39.204)1,344,168(34 p. 8 yd. 6 ft.
       1 176 12
       --------
       . 168 048
         156 816
       ---------
          11 232
             30¼
       ---------
         336 960
           2 808.
       ---------
         339 768(8 yd.
         313 632
       ---------
         . 26 136
                9
       ---------
         235 224(6 ft.
         235 224
       ---------
         ......0
```

I take 1,344,168, the given number of square inches, for a dividend, and for a divisor, the integer, 1 square perch reduced to the same denomination, square inches; that is, 39,204 square inches, and dividing as per rule 2d of compound division, I obtain 34 perches, 8 yards, 6 feet.

2. Reduce 662,709,600 seconds to years. 21 years.

3. " 40,356 square feet to square roods. 3r. 28p. 7yd.

4. Reduce 164,592 square inches to perches. 4 p. 6 yd.
5. " 624,580 yards to degrees. 5 deg. 7 m. 3 fur.

Case 2.

268. When both the dividend and divisor are complex, and the quotient like the dividend.

RULE 3.

269. 1st, Reduce the divisor to its lowest denomination; then, multiply the dividend by the numbers which were used to reduce the divisor; or, by their product, and then divide as in case first.

EXAMPLE.

1. What is the price of 1 yard of cloth, if 6 yards, 2 quarters, 1 nail, cost 34*l*. 5*s*. 9$\frac{3}{8}$*d*. ?

6yd. 2qr. 1na.)	34*l*. 5*s*. 9$\frac{3}{8}$*d*.
4	4
26	137 3 1$\frac{2}{4}$
4	4
105	105)548 12 6(5*l*.
	525
	23
	20
	105)472(4*s*.
	420
	52
	12
	105)630(6.
	630
	..0.

I, in the first place, reduce the divisor to nails, its lowest denomination; I then multiply the dividend by 4 and by 4, or by 16, and then having a *complex* dividend, to divide by an *incomplex* divisor, I perform the division as in case first, and find 5*l*. 4*s*. 6*d*. for the price of one yard.

2. I bought a cask of wine, containing 21 gallons, 3 quarts, 1 pint, for 43*l*. 14*s*. 8*d*. how much is it per gallon?
1*l*. 19*s*. 11$\frac{3}{4}$*d*. +

3. 10 gallons of molasses cost 2*l*. 12*s*. 6*d*.; how much is it per gallon? 5*s*. 3*d*.

4. 55 English ells, 3 quarters, being sold for 20*l.* 17*s.* 7½*d.*; how much is it per yard? 7*s.* 9½*d.*

5. If 99*l.* 3*s.* 11*d.* are paid for 33 hundred weight, 1 quarter of raisins; how much is it per hundred weight? 2*l.* 19*s.* 8*d.*

6. If 3 quarters, 26 pounds of sugar cost 4*l.* 11*s.* 8*d.*; what is it per pound? 10*d.*

7. 6*l.* 2*s.* is the price of 45 bushels, 2 pecks; how much is it per peck? 2*s.* 8*d.*

8. 33 hundred weight, 1 quarter, 22 pounds of iron being sold for 46*l.* 16*s.* 6*d.*; what is it per hundred weight? 1*l.* 8*s.*

9. If 74 bushels, 3 pecks of wheat are sold for 65 dollars, 78 cents; how much is it per bushel? 88 cents.

10. Paid 8 dollars, 46 cents, for 3 yards, 3 quarters of lace; how much is it per yard? $2.25, 6m.

11. Paid 320 dollars, 80 cents, for 17 hundred weight, 3 quarters, 17 pounds of tobacco; how much is it per ounce? 1ct.

12. A merchant asks 2 dollars, 25 cents a gallon for brandy, per the single gallon; but he says he will give me a cask containing 27 gallons, 2 quarts, 1 pint, for 54 dollars, 94½ cents cash; how much will it be per gallon? $1.98

CASE 3.

270. When both the dividend and divisor are complex, and the quotient is not of the same kind with the dividend; as in this case the dividend and divisor must be alike, work by

RULE 4.

271. Reduce both the dividend and divisor to the least denomination they contain, and then proceed as in the second rule of first case (art. 266.)

EXAMPLE.

1. How many yards of cloth can I have for 37*l.* 10*s.* at the rate of 1*l.* 14*s.* 6½*d.* per yard? 21 yd. 2 qr. 3 na.

1*l.* 14*s.* $6\frac{1}{2}$*d.*) 37*l.* 10*s.*
20 20
—— ——
34 750
12 12
—— ——
414. 9000
2 2
—— ——
829)18000(21yd.
1658
——
.1420
829
——
.591
4
——
829)2364(2 qr.
1658
——
.706
4
——
829)2824.(3 nails.
2487
——
.337

Since the dividend is in pounds, shillings, &c. and since I wish to have yards in the quotient, they are unlike each other; and as the dividend and divisor are alike, I reduce them both to their lowest denomination, which in this case expresses $\frac{1}{2}$. This reduction being performed, I have 18,000 to divide by 829, which gives 21 yards for quotient, and a remainder of 591. I reduce it to quarters, and obtain 2 for quotient, and then reduce the remainder 706, into nails, and get 3 nails in the quotient, and a remainder of 337, which I entirely neglect.

2. How many gallons of brandy can be bought for 54*l.* 17*s.* 6*d.* at the rate of 14*s.* 8*d.* (Pennsylvania currency)per gallon? 74 gal. 3 qt. +

3. A merchant has 174 dollars, 48 cents, to buy silk stockings with; how many pair can he buy, at the rate of 2 dollars, $62\frac{1}{2}$ cents per pair? 66 pair, and 1 dollar, 23 cents over.

4. How much land can be bought for 245*l.* 14*s.* 10*d.* at the rate of 19*l.* 19*s.* 6*d.* per acre? 12 a. 1 r. 8 p. +

5. With 20*l.* 14*s.* 7*d.* how many pounds of candles can I buy, at the rate of $14\frac{1}{2}$*d.* per pound? 343 lb.

6. With 78 dollars, 46 cents, how many bushels of dried peaches can I buy, at the rate of 2 dollars, $12\frac{1}{2}$ cents per bushel? 36 bu. 3 pk. 5 qt. 1 pt.

7. How much hay can I purchase for 548 dollars, 16 cents, at the rate of 78 cents per cwt.? 702 cwt. 3 qr. 2 lb.

APPENDIX.

NUMERATION.

Remark I. (See page 5.) The federal money of the United States, presents a complete and admirable system of numeration, which may, in case the explanation already given should not be sufficient, serve to explain the formation of the different classes. For, 10 cents are equal to 1 small silver piece, called a *dime:* 10 of those dimes to a dollar; and, 10 dollars to an eagle. An arrangement, in which the ten-fold gradation is invariably observed, and which is by far preferable to the subdivision of the dollar into sixteenths, eighths, &c.

Any pupil a little acquainted with federal money, will readily answer the following questions. If you have 10 cents, can you exchange them for 1 single silver piece? Yes: for a *ten-cent* piece or a *dime.* If you get 10 dimes, can you not exchange them for 1 larger silver piece? Yes: for one *dollar.*

Then, he immediately and easily perceives the possibility of converting 10 units of one denomination, into a single one of another. He has, then, overcome the greatest difficulty, which the system of numeration presents to beginners. There is yet another thing to be understood, viz: the necessity of separating the different classes, of giving to each its proper and allotted rank. It is, however, explained and illustrated as follows:

Suppose you have three small boxes; the first or right hand one, can contain but 9 cents; the second, only 9 dimes; the third, only 9 dollars.

3d class.	2d class.	1st class.
—	—	—
dollars.	dimes.	cents.
0	0	9
		1
0	1	0
		9
0	1	9
		3
0	2	2
	1	8
0	4	0
	6	3
1	0	3

If you receive 9 cents, your first, or cent box is full. Suppose you receive 1 cent more, as it cannot be put in your cent box, what will you do? Will it not be better to take the 9 cents out of the box, and adding the tenth to them, exchange the whole for a single silver piece? Yes. And, where will you put that silver piece? In the cent box or dime box? In the dime or second box. What will then be in your boxes? 1 silver piece in the second, and nothing in the first and third. Now please to set down under each box what is in it. After it is done; ask, what number is that?—Ten.

If now you receive 9 cents, where will you put them? In the cent box or dime box? In the cent box; and then I shall have 1 silver piece in the second box, and 9 cents in the first. Set it down, if you please. When done: how many cents altogether? Nineteen. That is, 1 unit of the second class, and 9 of the first.

If, now, 3 cents more be given to you, you will have 12; and, if you exchange 10 of them for another silver piece, where will you put it? With the first one, in the dime box. What then will there be in each box? 2 silver pieces in the second, and 2 cents in the first. Written 22—that is, 2 units of the first order, and 2 of the second.

If you now receive 18 cents, as 9 only can go into your cent box, how will you manage? Exchange 10 of them for 1 silver piece, and put it in the dime box, with the 2 dimes I have there already: then put the remaining 8 cents in the cent box. But stop, I find 2 in it; I have, of course, 10 cents more, and as they cannot all go into the cent box, I will exchange them for another dime, and adding it to the 3, which are already in the second box, there will be 4 dimes in it, and 0 in the cent box. Written 40—forty—4 units of the second class and none of the first.

Now, suppose you receive 6 dimes and 3 cents, how will you place them? Put the 3 cents in the cent box, and as I

shall have 10 dimes which cannot go into the dime box, will exchange them for 1 dollar, and place it in the third bo: 1 shall then have 1 dollar, 0 dime, and 3 cents, represente by 103—one hundred and three. 1 unit of the third orde none of the second, and 3 of the first, &c.

These practical exercises may be varied at pleasure, an shortened or lengthened according to the degree of impres sion they make on the pupil.

To superficial observers, these details may appear long tedious and trivial, but they cannot be viewed in that ligh by the philosophical observer, who has attentively regarde the slow progress of juvenile improvement. He who wishe to impart a competent knowledge of the science of number must be careful to impress strongly, the system of numera tion on the minds of his pupils: for, this is the basis of al subsequent operations. For want of this indispensable re quisite, how many scholars are there, who, although they ca readily perform the different operations of arithmetic, fin themselves at a loss to point off the decimals, and are, in fac unable to do it, when the answer is not subjoined to th question they have to solve.

DIVISION.—(See page 35.)

The method here recommended is not new, it is frequentl employed in long operations; such as laying of taxes, contri butions, &c. I introduce it as worthy of attention.

When the pupil comes to division, being already acquainte with subtraction and multiplication, he is supposed capabl of performing these two operations with ease; the only ob stacle he meets with, is the difficulty of finding out the pro per figures of the quotient, and that difficulty is frequentl discouraging. To remove this stumbling block, the teache may with success adopt the following method, which we wil explain in working an example or two.

Divide 3240458, by 5.

1st, I multiply the divisor 5, successively by 1, by 2, by 3 4, 5, 6, 7, 8 and 9, and obtain 9 products, or multiples of the divisor, as follows:

Then I place the dividend and divisor in this manner:

Divisor. Dividend. Quotient.

	$\cdot\cdot\cdot\cdot\cdot\cdot\cdot$
$5\times1=5$	5) 3240458. (648091
$2=10$	30
$3=15$	24
$4=20$	20
$5=25$	40
$6=30$	40
$7=35$	.045
$8=40$	45
$9=45$	.08
	5
	3

and taking 32 for first dividual, I perceive that the highest product it contains is 30, which comes from the multiplying of the divisor 5, by 6; I then set down 6 in the quotient, and placing the product 30, under the dividual 32, I subtract and find 2 for remainder. Then putting a dot on the 4, the next figure of the dividend, I bring it down, and place it on the right of the remainder 2, so as to have 24, and seeing that the highest product contained in this second dividual is 20, which comes from the multiplication of 5 by 4, I place 4, as the second quotient figure, and subtracting 20 from 24, I find a second remainder of 4. This last remainder increased by 0, the next figure of the dividend, (over which I place a dot, to show, at once, how far I have proceeded,) becomes the third dividual 40, which containing the product 40, equal to 8 times 5, gives 8 for the third quotient figure, and leaves no remainder to the subtraction. I next bring down the 4 of the dividend for the fourth dividual, and as this dividual 4, does not contain the divisor 5, even once, I say it contains it 0 times, and accordingly place 0 for the fourth quotient figure, and having nothing to subtract, I immediately bring down 5, the next figure of the dividend (dotting it, as well as the others,) on the right of the 4, which gives me 45 for fifth dividual. This dividual gives 9 for the fifth quotient figure, and the subtraction being without a remainder, I bring down the 8, last figure of the dividend, for sixth dividual, which gives 1 for the sixth quotient figure, and leaves a remainder of 3. Hence, the final quotient is 648091 and $\frac{3}{5}$; for, to complete the quotient, place the last remainder to the end of it,

above a line, and write the divisor below, thus; $\frac{3}{5}$, which means 3 fifths.

This method is quickly understood, and the operation easily performed by the pupil, even when he is unable to do it in the usual way, and, on that account, deserves some attention. But it has another advantage, viz: that of answering when the divisor is less than 12, as well as when it is more; and by that means, it supercedes the use of *short* and *long* division. These two operations, although they are founded on the same principle, and although they are virtually the same, are considered by beginners, on account of their being performed in two different ways, as entirely distinct; and they are, for that reason, perplexing.

Divide 842,304,765 by 185.

The first thing I have to do, as well in this as in every other division, is to form the table of multiples of the divisor, as follows:

Table.	
185×1=	185
2=	370
3=	555
4=	740
5=	925
6=	1110
7=	1295
8=	1480
9=	1665

Divisor. Dividend. Quotient.

```
       .......
185.)842304765(4552998+135/185
     740
     ---
     1023
      925
     ----
     ..980
       925
       ---
       .554
        370
        ---
       .1847
        1665
        ----
         1826
         1665
         ----
         .1615
          1480
          ----
           135
```

and then a single glance shows me which is the highest product contained in each new dividual; and consequently, opposite to it, I find the quotient figure, without any further trouble.

It may be objected that when the divisor is less than 12, as all the multiples are in the range of the multiplication table, they ought to be sufficiently familiar to the pupil's mind to save him the troublesome task of setting them down. To this objection, I offer the following reply. 1st, If the multiplication table be very familiar to the student, the trouble of setting down the multiples is but trifling. 2d, If the multiplication table be *not* familiar, no exercise could be better calculated to impress it on his mind. 3d, I know, from experience, that the time employed in forming the table of multiples, is more than regained by the facility it gives in performing the operation. It must be plain that when they are once down, the student having only mere subtractions to perform, and being able, at one glance, to select the right subtrahend, he may proceed as expeditiously as he pleases, without being, at every step, stopped by the *trials* requisite for every new quotient figure; trials, which are always difficult and tedious, and which consume a great deal of time.

DIVISION.—(See page 41.)

To make any quantity a certain number of times *greater*, you must multiply it by that number; and, on the contrary, *divide* it, if you intend to make it less. Hence, to make the product of a multiplication 2, 3, 4, &c. times *greater*, one of its factors must be *multiplied* by 2, 3, 4, &c. or *divided*, if it be required to make it *less*.

The product does not change when one of its factors is multiplied by a certain number, if at the same time, the other factor is *divided* by the *same* number.

The quotient *increases* when the dividend is *multiplied*, and *decreases* when the dividend is *divided;* and, on the contrary, the *quotient diminishes when the divisor is multiplied*, and *augments when the divisor is divided.*

The value of the quotient remains the same, if both the dividend and divisor, are multiplied or divided by the same number.

Divide 1 by 1, the quotient is 1—10 by 10, the quotient is 1 yet—50 by 5, the quotient is 10—10 by 1, the quotient is 10, the same.

PROPORTIONS.—(See page 63.)

We can, not only, find the first term of a proportion, but any one of them, provided three are given. For among those *three, if two* of them are *means*, the other is an *extreme*, and

in this case, the product of the *means*, divided
extreme, will give the other *extreme*.

Ex.—$7 : 56 :: 42 : x$.* $56 \times 42 = 2352$ product
which being divided by 7, the known extreme
the second *extreme*.

$x : 14 :: 64 : 448$	$12 : 156 ::$
$2 : 64 :: 14 : x$	$108 : 18 :: 2$
$48 : 384 :: 7 : x$	$x : 246 ::$

If two of the terms are *extremes*, the third
means, and, in this case, the product of the *ex*
by the known *means*, will give the other mean

Ex.—$17 : x :: 63 : 315$. $315 \times 17 = 5355$ pro
tremes, which being divided by the known m
$85 = x$, the other means.

$14 : x :: 448 : 64$	$168 : 12 ::$
$64 : 2 :: x : 14$	$294 : x ::$
$28 : x :: 228 : 57$	

FRACTIONS.—(See page 88.)

To find the greatest common divisor, or g
measure of a fraction.

RULE.

Divide the greater term by the less, then t
first remainder; afterwards, that first remai
cond; then, the second remainder by the th
until nothing remain. The last divisor, will
common divisor required.

If that last divisor is 1, it is a proof that th
not be simplified.

Ex. What is the greatest common divisor c

```
              235)564(2
                  470
                  ---
1st remainder 94)235(2
                 188
                 ---
2d remainder and }  47)94(2
  last divisor.  }     94
                       --0
```

47 being the
the greatest co
and to prove i
both terms of t
by 47, as below

```
Thus, 47)235(5 numerator.     47)564(12 
         235                     47
         ---                     ---
           0                      94
                                  94
                                  --0
```

Fraction reduced $\frac{5}{12}$

2. Reduce $\frac{1377}{3672}$ to its lowest terms.

3. Required the common measure of $\frac{475}{589}$.

The reason is this: any number which measures two others, will also measure their *sum*, their *difference*, and consequently, any *multiple* of either.

4. Reduce $\frac{268}{436}$ to its lowest terms.

5. " $\frac{299}{481}$ "

6. " $\frac{374}{418}$ "

7. " $\frac{1751}{1819}$. "

RULE OF THREE.—(See page 72.)

RULE FOR STATING.

Set that term of the supposition which is homogeneous with the term of demand, in the first place; set the other term of supposition in the second, and the term of demand in the third place.

When the question is thus stated, consider whether the proportion is direct or inverse.

The proportion is direct, when the third term is greater than the first, and the nature of the question requires that the fourth term, or answer, should be greater than the second; or when the third term is less than the first, and it is required that the fourth term be less than the second.

The proportion is inverse, when the third term is greater than the first, and the fourth is to be less than the second; or when the third term is less than the first, and the fourth is to be greater than the second.

RULE FOR DIRECT PROPORTION.

If the first and third terms be not of the same denomination, reduce both to the lowest in either; and if the second term consist of several denominations, reduce it to its lowest denomination: then, multiply the second and third terms together, and divide the product by the first term; the quotient will be the fourth term, or answer, in the same denomination as the second, or that to which the second was reduced.

EXAMPLE.

If 2 yards of muslin cost 4s. what will 6 yards cost?

```
yd.  s.   yd.
 2 : 4 : : 6
           4
         ---
         2)24
         ---
           12 Answer.
```

RULE FOR INVERSE PROPORTION.

Multiply the first and second terms together, and divide the product by the third; the quotient will be the answer in the same denomination as the second, or that to which the second was reduced.

EXAMPLE.

If 4 men can build a wall in 4 days, how many men can do it in 8 days?

```
men.  days.  days.
 4  :  4  : :  8
       4
      ---
     8)16
      ---
        2 Answer.
```

COMPOUND RULE OF THREE.

Write the terms of the supposition which are similar to those of the demand, under one another, in the first place; in the second, set that of the same kind as the answer; and, in the third, each demanding term correspondent and similar to its supposition, on the same line with it.

When the question is stated, consider the two upper terms with the middle one, as a stating in the single rule of three; then, the two next terms with the same middle one, as another stating in the single rule of three, and so on, of every other set of terms.

If all the statements are direct, then the question is in direct proportion; but if one of the statements is inverse, the question is inverse.

RULE FOR DIRECT PROPORTION.

Multiply all the first terms together for a new first term; all the third terms together for a new third term: then, the

statement becomes a single rule, and having only three terms to find the fourth, multiply the second by the third, and divide the product by the first. The quotient will be the answer in the same denomination with the middle term.

NOTE.—Simplify or cancel if possible, before multiplication.

EXAMPLE.

If 6 men in 8 days eat 10 pounds of bread, how much will 12 men eat in 24 days?

$$\left.\begin{array}{l}6 \text{ men}\\ 8 \text{ days.}\end{array}\right\} 10 \text{ lb.} \left\{\begin{array}{l}12 \text{ men}\\ 24 \text{ days.}\end{array}\right.$$

```
     48
     24
    ---
    288
     10
    ---
48)2880(60 lb.
   288
   ---
     0
```

Contracted.

$$\left.\begin{array}{l}\cancel{6}\\ \cancel{8}\end{array}\right\} 10 \left\{\begin{array}{ll}\cancel{12} & 2\\ \cancel{24} & 3\end{array}\right.$$

```
 6
10
--
60 lb.
```

RULE FOR INVERSE PROPORTION.

Transpose the inverse terms; that is, set that which is in the first place, under the third; and that which is in the third place under the first; then work as in direct proportion.

EXAMPLE.

If 7 men reap 84 acres of wheat in 12 days, how many men can reap 100 acres in 5 days?

$$\left.\begin{array}{r}84 \text{ A}\\ \cancel{12} \text{ D.}\\ 5\end{array}\right\} 7 \text{ m.} \left\{\begin{array}{r}100 \text{ direct}\\ \cancel{5} \text{ inverse}\\ 12\end{array}\right.$$

```
420        1200
              7
           ----
     420)8400(20
         840
         ---
           0
```

Contracted.

$$\left.\begin{array}{r})\cancel{84}\\ \cancel{12}\\ \cancel{5}\\ \cancel{7}\end{array}\right\} \cancel{7} \text{ m.} \left\{\begin{array}{l}\cancel{100}(20 \text{ Ans.}\\ \cancel{5}\\ \cancel{12}\end{array}\right.$$

Rupee 50 cts. (lowest 45 highest 56¼) Pie 12th of Ana —
Lac of Rupees 50 000 dollars — Ana 3 cts. Indigo Saltpetre
16 Anas a Rupee — 74 lbs) used for & some others
Maund (factory
... Maund 82 lbs — used for everything else —

Every Dec. Fraction has a Denominator understood viz. unity or one & as many Cyphers as there are decimal places.

$\frac{1}{10} = .1$

$\frac{1}{100} = .01$

$\frac{1}{1000} = .001$

$\frac{1}{1000000} = .000001$

$\frac{282}{1000000} = .000282$

$\frac{2463}{100000} = .02463$

$.02506 = \frac{2506}{100000}$

$.000002 = \frac{2}{1000000}$

$.2 + .031 + .00217$

$$\begin{array}{r} .2 \\ .031 \\ .00217 \\ \hline .23317 \end{array}$$

$.0014 - .0009$

$$\begin{array}{r} .0014 \\ .0009 \\ \hline .0005 \end{array}$$

$\frac{1}{100} \times \frac{10}{10} = .010$

$\frac{1}{100} \times \frac{1000}{1000} = \frac{1000}{100000} = .0010$

$.013 \times .000231$

$\frac{13}{1000} \times \frac{231}{1000000} = \frac{3003}{1000000000} = .000003003$

$$\begin{array}{r} .013 \\ .000231 \\ \hline 13 \\ 39 \\ 26 \\ \hline .3003 \end{array}$$

Computing Interest

Editors Traveller:—I observed an article on the fourth page of your paper, a few days ago, representing that a correspondent of the Baltimore Sun had learned, *twelve* years ago, a simple plan for computing interest at six per cent. Almost all persons who have any amount of interest to cast, have more or less short-hand methods of obtaining their results, and by using less figures than are used in the ordinary method. It must be obvious, therefore, that if the principle be correct, any short-hand way of casting interest, discount, or commissions, is the saving of so much time; and this is a great thing with the book-keeper. For the past twenty-five years, I have had in use a short-hand method of figuring; and, hoping it may be of some use to the "profession," I will describe it. In casting interest for days, at six per cent., multiply by the number of days and divide by 6. This is much simpler than the plan described in the Baltimore Sun. To ascertain the interest for the three days grace on notes, if the amount is less than one hundred dollars, take half of the first left-hand figure; if over a hundred dollars, take half of the first *two* left-hand figures; if thousands, half of the first *three* figures, &c. Thus, what is the interest of the grace (or three days) on $50? Answer, 2 cents 5 mills, or 2½ cents; on $500, 25 cents; on $5000, $2.50, and so on. Also, in casting commissions, at 2½ per cent., it is not necessary to multiply by the amount, but simply divide by *four*, and you will get the answer correctly; and the same rule could be applied to interest. Thus, what is the interest of $300 for five months, at six per cent. Of course it is ½ per cent. per month, or 2½ per cent—stated thus: $300, 5 months, divided by 4, gives $7,50. By these very simple methods, the sums given in the "Sun" could be done in an incredibly short time. Thus, he says, what is the interest of $100 for 21 days? 3 days is 5 cents, 21 days is 7 times as much, or 35 cents. Again, what is the interest of $378 for 93 days? 30 days is ½ per cent., $1,89; 90 days is 3 times 30, or 5,67; 3 days is half of the first two figures, or 18 9-10 cents, $5,85 9-10. Let book keepers try these rules, (if they do not already know them), and much saving of time will be experienced.

AN OLD BOOK-KEEPER.